Non-smooth simulation of a 6-DOF dynamical model of the grand piano action

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Introduction   The grand piano action has been developped empirically and provides a remarkably accurate control of the hammer velocity and its impact time [1]. It is made of seven rotating bodies (Fig. 1, left) with parallel axes and felts at contact zones. The numerical simulation presented here aims at understanding its dynamics in view of improving numerical keyboards and contributing to knowledge on haptic controllability.
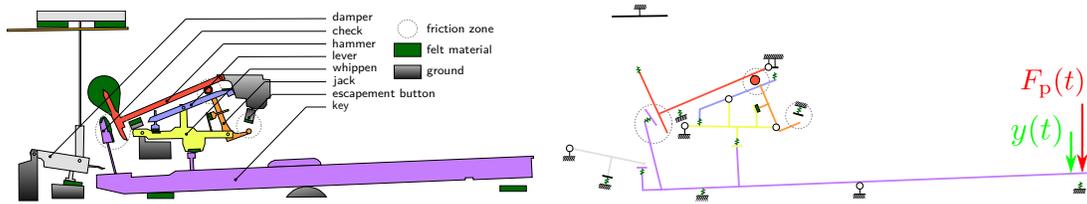
Figure 1. Left: scheme of the grand piano action. Right: rigid multibody model. $F_\text{p}(t)$ is the force exerted/felt by the player, $y(t)$ is the displacement of the key.

Simulation input   Simulating the mechanism may consist in computing the motion of the key in response to a given force (e.g. [2]) or vice versa. Because the inertia dominates the dynamics of the action, the effects of its other dynamical features are smoothened in force-driven simulations; this can be observed by means of an elementary 1-DOF model. The simulations presented here are driven by the measured position $y(t)$.

Non-smooth simulation   We used a model based on that proposed by Lozada [3]. The 7 bodies are considered as 6 rotating solids with dry and viscous friction on their axes and 13 non-linear and localized springs representing the felts (Fig. 1, right). Any spring force is generically given by $F(\delta) = k\,\delta^r + b\,\delta^2\,\dot{\delta}$, where $\delta > 0$ is the length of compression of the spring (felt). The equation describing the dynamics are given the generic form (1), where the tangential Coulomb friction is omitted for simplicity.

$$\mathbf{M}(\mathbf{x})\,\ddot{\mathbf{x}} + \mathbf{N}(\mathbf{x},\dot{\mathbf{x}})\,\dot{\mathbf{x}} + \mathbf{c}_\text{v}\,\dot{\mathbf{x}} + \mathbf{c}_\text{d}\,\mathbf{sign}(\dot{\mathbf{x}}) + \left(\frac{\partial\delta}{\partial\mathbf{x}}(\mathbf{x})\right)^T \mathbf{F}(\delta) + \mathbf{F}^\star(\mathbf{x},t) = \mathbf{0} \qquad (1)$$

$\mathbf{x}$ is the vector of generalized coordinates (*i.e.* the 6 angles), $\mathbf{M}$ is the mass matrix, $\mathbf{N}$ gathers the non-linear dynamic terms, $\mathbf{c}_\text{v}$ and $\mathbf{c}_\text{d}$ are diagonal matrices of the viscous and the dry joint friction coefficients respectively, δ is the vector of the lengths of compression of the springs and $\mathbf{F}^\star$ is the vector of all the smooth forces which are not contact forces nor related to joint friction, such as gravity or F_p. As usual, **sign** is the vector of the set-valued functions sign defined by $\text{sign}(\dot{\theta}) = \dot{\theta}/|\dot{\theta}|$ if $\dot{\theta} \neq 0$ and the whole set

$[-1,1]$ if $\dot{\theta} = 0$ so that the dry friction is described by the Coulomb model. This implies that Eq. (1) is not an ODE. Regularizing the sign set-valued functions yields ODEs. However, a reasonable precision may require a time step too small for practical use. Another difficulty is that stick-slip transitions induce velocity singularities. These difficulties are efficiently overcome by using methods of non-smooth contact dynamics (NSCD). Instead of writing the dynamics as six coupled equations of the form (1), we use a Measure Differential Inclusion formulation [4, 5] written here as:

$$\begin{cases} \mathbf{M}(\mathbf{x})\,d\mathbf{v} \ = \mathbf{F}^{\star}(\mathbf{x},\dot{\mathbf{x}},t)\,dt + \mathbf{H}(\mathbf{x})\,d\mathbf{i} \\ \mathbf{v}^{+} \ = (\dot{\mathbf{x}})^{+} \\ (\mathbf{g}(\mathbf{x}),\ \mathbf{H}^{T}(\mathbf{x}).\mathbf{v}^{+},\ d\mathbf{i}) \in K \end{cases} \qquad (2)$$

The first equation formulates the non-smooth dynamics. $d\mathbf{v}$ and $d\mathbf{i}$ are vector-valued measures on $\mathbb{R}$ and can therefore be non-smooth. All the smooth terms, such as non-linear dynamic terms or viscous friction, are included in $\mathbf{F}^{\star}$. $\mathbf{H}^{T}$ yields the relative velocities in the contact frame as a function of the generalized velocities. The non-smooth laws (tangential Coulomb friction at contact points and joint friction) and equality constraints are written as an inclusion in the fixed set K.

Results Eqs. (2) are discretized using a time-stepping scheme and solved by means of an implicit scheme with a 0.5 ms time step. Results for two different keystrokes are presented in Fig. 2. The simulations are in very good agreement with the measurements. The small time-shift observed in the *piano* keystroke is probably due to a small discrepancy in the geometrical description, to which the force profile is very sensitive.
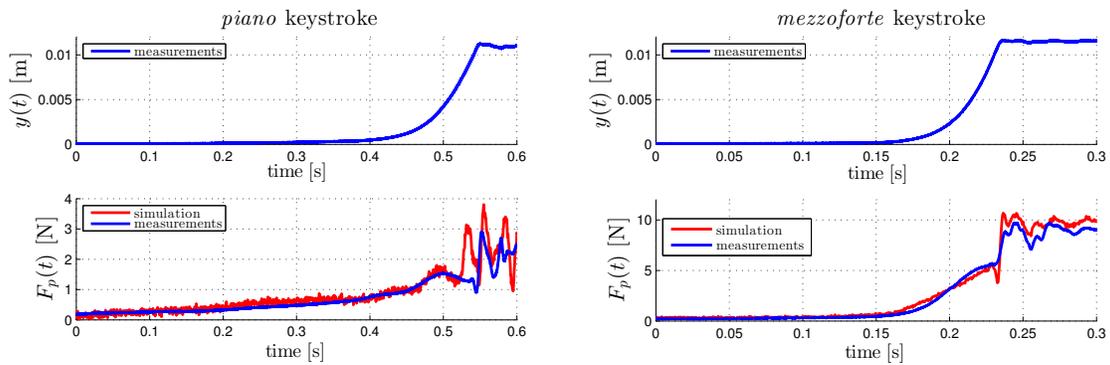
Figure 2. Top: measured position of the key $y(t)$, serving as an input for the simulation. Bottom: comparison between the simulated and the measured forces (see Fig. 1). Left: *piano* keystroke. Right: *mezzoforte* keystroke.

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