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Michel Fliess[a,c,*] and Cédric Join[b,c,d]

[a]*LIX (CNRS, UMR 7161), École polytechnique, 91128 Palaiseau, France;* [b]*CRAN (CNRS, UMR 7039), Université de Lorraine, BP 239, 54506 Vandœuvre-lès-Nancy, France;* [c]*AL.I.E.N. (ALgèbre pour Identification et Estimation Numériques), 24-30 rue Lionnois, BP 60120, 54003 Nancy, France;* [d]*Projet NON-A, INRIA Lille – Nord-Europe, France*

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'Model-free control'and the corresponding 'intelligent' PID controllers (iPIDs), which already had many successful concrete applications, are presented here for the first time in an unified manner, where the new advances are taken into account. The basics of model-free control is now employing some old functional analysis and some elementary differential algebra. The estimation techniques become quite straightforward via a recent online parameter identification approach. The importance of iPIs and especially of iPs is deduced from the presence of friction. The strange industrial ubiquity of classic PIDs and the great difficulty for tuning them in complex situations is deduced, via an elementary sampling, from their connections with iPIDs. Several numerical simulations are presented which include some infinite-dimensional systems. They demonstrate not only the power of our intelligent controllers but also the great simplicity for tuning them.

Keywords: Model-free control; PID controllers; intelligent PID controllers; intelligent PI controllers; intelligent P controllers; estimation; noise; flatness-based control; delay systems; non-minimum phase systems; fault accommodation; heat partial differential equations; operational calculus; functional analysis; differential algebra

1. Introduction

Although *model-free control* was introduced only a few years ago (Fliess & Join, 2008a, 2009; Fliess, Join, & Riachy, 2011b), there is already a quite impressive list of successful concrete applications in most diverse fields, ranging from intelligent transportation systems to energy management (Abouaïssa, Fliess, Iordanova, & Join, 2012; Andary, Chemori, Benoit, & Sallantin, 2012; Choi, d'Andréa-Novel, Fliess, Mounier, & Villagra, 2009; d'Andréa-Novel et al., 2010; De Miras, Riachy, Fliess, Join, & Bonnet, 2012; Formentin, de Filippi, Corno, Tanelli, & Savaresi, 2013; Formentin, de Filippi, Tanelli, & Savaresi, 2010; Gédouin et al., 2011; Join, Masse, & Fliess, 2008; Join, Robert, & Fliess, 2010a, 2010b; Lafont, Pessel, Balmat, & Fliess, 2013; Michel, Join, Fliess, Sicard, & Chériti, 2010; Milanes, Villagra, Perez, & Gonzalez, 2012; Sorcia-Vázquez, García-Beltrán, Reyes-Reyes, & Rodríguez-Palacios, 2010; Villagra, d'Andréa-Novel, Choi, Fliess, & Mounier, 2009; Villagra & Balaguer, 2011; Villagra & Herrero-Pérez, 2012; Villagra, Milanés, Pérez, & de Pedro, 2010; Wang, Mounier, Cela, & Niculescu, 2011). Most of those references lead to practical implementations. Some of them are related to patents.

Remark 1: The wording *model-free control* is of course not new in the literature, where it has already been employed by a number of authors. The corresponding literature is huge: see Bilal Kadri (2009), Chang, Gao, and Gu (2011), Hahn and Oldham (2012), Hong-wei, Rong-min, and Hui-xing (2011), Keel and Bhattacharyya (2008), Killingsworth and Krstic (2006), Malis and Chaumette (2002), dos Santos Coelho, Wicthoff Pessôa, Rodrigues Sumar, and Rodrigues Coelho (2010), Spall and Cristion (1998), Swevers et al. (2007), Syafiie, Tadeo, Martinez, and Alvarez (2011), and Xu, Li, and Wang (2012). The corresponding settings are quite varied. They range from 'classic' proportional-integral-derivative (PIDs) to robust and adaptative control via techniques stemming from, e.g., neural nets, fuzzy systems, and soft computing. To the best of our understanding, those approaches are rather far from what we are developing here. Let us emphasise, however, Remark **5** for a comment on some works which are perhaps closer. See also Remark **3**.

Let us now summarise some of the main theoretical ideas which are shaping our model-free control. We restrict ourselves for simplicity's sake to systems with a single control variable u and a single output variable y. The unknown 'complex' mathematical model is replaced by an *ultra-local model*

$$\boxed{y^{(\nu)} = F + \alpha u} \tag{1}$$

(1) $y^{(\nu)}$ is the derivative of order $\nu \geq 1$ of y. The integer ν is selected by the practitioner. The existing

*Corresponding author. Email: Michel.Fliess@polytechnique.edu

examples show that ν may always be chosen quite low, i.e., 1, or, only seldom, 2. See Section 4 for an explanation.

(2) $\alpha \in \mathbb{R}$ is a non-physical constant parameter. It is chosen by the practitioner such that αu and $y^{(\nu)}$ are of the same magnitude. It should be therefore clear that its numerical value, which is obtained by trials and errors, is not *a priori* precisely defined. Let us stress moreover that controlling industrial plants has always been achieved by collaborating with engineers who know the system behaviour well.

(3) F, which is continuously updated, subsumes the poorly known parts of the plant as well as of the various possible disturbances, without the need to make any distinction between them.

(4) For its estimation, F is approximated by a piecewise constant function. Then the algebraic identification techniques due to Fliess and Sira-Ramírez (2003, 2008) are applied to the equation

$$y^{(\nu)} = \phi + \alpha u, \tag{2}$$

where ϕ is an unknown constant parameter. The estimation

- necessitates only a quite short time lapse,
- is expressed via algebraic formulae which contain low-pass filters like iterated time integrals,
- is robust with respect to quite strong noise corruption, according to the new setting of noises via *quick fluctuations* (Fliess, 2006).

Remark 2: The following comparison with computer graphics might be enlightening. Reproducing on a screen a complex plane curve is not achieved via the equations defining that curve but by approximating it with short straight line segments. Equation (1) might be viewed as a kind of analogue of such a short segment.

Remark 3: Our terminology *model-free control* is best explained by the ultra-local Equation (1) which implies that the need of any 'good' and 'global' modelling is abandoned.

Assume that $\nu = 2$ in Equation (1):

$$\ddot{y} = F + \alpha u. \tag{3}$$

Close the loop via the *intelligent proportional-integral-derivative controller*, or *iPID*,

$$\boxed{u = -\frac{F - \ddot{y}^* + K_P e + K_I \int e + K_D \dot{e}}{\alpha}} \tag{4}$$

where

- y^* is the reference trajectory,
- $e = y - y^*$ is the tracking error,
- K_P, K_I, K_D are the usual tuning gains.

Combining Equations (3) and (4) yields

$$\ddot{e} + K_D \dot{e} + K_P e + K_I \int e = 0. \tag{5}$$

Note that F does not appear anymore in Equation (5), i.e., the unknown parts and disturbances of the plant vanish. We are, therefore, left with a linear differential equation with constant coefficients of order 3. The tuning of K_P, K_I, and K_D becomes, therefore, straightforward for obtaining a 'good' tracking of y^*. This is a major benefit when compared to the tuning of 'classic'PIDs.

Remark 4: *Intelligent PID controllers* may already be found in the literature but with a different meaning (see Åström, Hang, Persson, & Ho, 1992).

Remark 5: See Chang and Jung (2009), Han (2009), Youcef-Toumi and Ito (1990), and Zheng, Chen, and Gao (2009) for some remote analogy with our calculations. Those references assume, however, that the system order is finite and moreover known.

Our paper is organised as follows. The general principles of model-free control and of the corresponding intelligent PIDs are presented in Section 2. The online estimation of the crucial term F is discussed in Section 3. Section 4 explains why the existence of frictions permits to restrict our intelligent PIDs to intelligent proportional or to intelligent proportional-integral correctors. The numerical simulations in Section 5 examine the following case-studies:

- A part of the unknown system may be nevertheless known. If it happens to be *flat* (Fliess, Lévine, Martin, & Rouchon, 1995; Lévine, 2009; Sira-Ramírez & Agrawal, 2004), it will greatly facilitate the choice of the reference trajectory and of the corresponding nominal control variable.
- Standard modifications including aging and an actuator fault keep the performances, with no damaging, of our model-free control synthesis without the need of any new calibration.
- An academic nonlinear case study demonstrates that a single model-free control is sufficient, whereas many classic PIDs may be necessary in the usual PID setting.
- Two examples of infinite-dimensional systems demonstrate that our model-free control provides excellent results without any further ado:

- a system with varying delays,
- a one-dimensional semi-linear heat equation, which is borrowed from Coron and Trélat (2004).

- A peculiar non-minimum phase linear system is presented.

Following d'Andréa-Novel, Boussard, Fliess, Join, Mounier, and Steux (2010), Section 6 explains the industrial capabilities of classic PIDs by relating them to our intelligent controllers. This quite surprising and unexpected result is achieved for the first time to the best of our knowledge. Section 7 concludes not only by a short list of open problems but also with a discussion of the possible influences on the development of automatic control, which might be brought by our model-free standpoint.

The appendix gives some more explanations on the deduction of Equation (1). We are employing

- rather old-fashioned functional analysis, which goes back to Volterra (1910, 1930) and Volterra and Pérès (1936). Note that this functional analysis is a mainstay in engineering since the introduction of *Volterra series* (see Barrett, 1963);
- some elementary facts stemming from differential algebra (Kolchin, 1973), which has been quite important in control theory since the appearance 20 years ago of *flatness-based control* (Fliess et al., 1995).

2. Model-free control: general principles

Our viewpoint on the general principles on model-free control was developed in Fliess, Join, and Sira-Ramírez (2006b), Fliess, Join, Mboup, and Sira-Ramírez (2006a), Fliess and Join (2008a, 2008b), Fliess and Join (2009), Fliess, Join, and Riachy (2011a), Fliess et al. (2011b).

2.1. Intelligent controllers

2.1.1 Generalities

Consider again the ultra-local model (1) . Close the loop via the *intelligent controller*

$$u = -\frac{F - y^{*(\nu)} + \mathfrak{C}(e)}{\alpha}, \tag{6}$$

where

- y^* is the output reference trajectory;
- $e = y - y^*$ is the tracking error;
- $\mathfrak{C}(e)$ is a *causal*, or *non-anticipative*, functional of e, i.e., $\mathfrak{C}(e)$ depends on the past and the present, and not on the future.

Remark 6: See Volterra (1910, 1930) and Volterra and Pérès (1936) for an intuitive and clever presentation of the early stages of the notion of *functionals*, which were also called sometimes *line functions*. See Section A.1 in the appendix for more details.

Remark 7: Imposing a reference trajectory y^* might lead, as well known, to severe difficulties with non-minimum phase systems: see Fliess and Marquez (2000), Fliess, Sira-Ramírez, and Marquez (1998), and Sira-Ramírez and Agrawal (2004) from a flatness-based viewpoint (Fliess et al., 1995; Sira-Ramírez & Agrawal, 2004). See also Remarks **9**, **20**, and Section 7.

Combining Equations (1) and (6) yields the functional equation:

$$e^{(\nu)} + \mathfrak{C}(e) = 0.$$

$\mathfrak{C}$ should be selected such that a perfect tracking is asymptotically ensured, i.e.,

$$\lim_{t \to +\infty} e(t) = 0. \tag{7}$$

This setting is too general and might not lead to easily implementable tools. This shortcoming is corrected below.

2.1.2 Intelligent PIDs

Set $\nu = 2$ in Equation (1). With Equation (3) define the intelligent proportional-integral-derivative controller, or iPID, (4). Combining Equations (3) and (4) yields Equation (5), where F does not appear anymore, i.e., the unknown parts and disturbances of the plant are eliminated. The tracking condition expressed by Equation (7) is, therefore, easily fulfilled by an appropriate tuning of K_P, K_I, and K_D. It boils down to the stability of a linear differential equation of order 3, with constant real coefficients. If $K_I = 0$ we obtain an *intelligent proportional-derivative controller*, or *iPD*:

$$\boxed{u = -\frac{F - \ddot{y}^* + K_P e + K_D \dot{e}}{\alpha}} \tag{8}$$

Assume now that $\nu = 1$ in Equation (1):

$$\boxed{\dot{y} = F + \alpha u} \tag{9}$$

The loop is closed by the *intelligent proportional-integral controller*, or *iPI*:

$$\boxed{u = -\frac{F - \dot{y}^* + K_P e + K_I \int e}{\alpha}} \tag{10}$$

Quite often K_I may be set to 0. It yields an *intelligent proportional controller*, or *iP*:

$$\boxed{u = -\frac{F - \dot{y}^* + K_P e}{\alpha}} \tag{11}$$

Results in Sections 4 and 6 explain why iPs are quite often encountered in practice. Their lack of any integration of the tracking errors demonstrate that the anti-windup algorithms, which are familiar with 'classic' PIDs and PIs, are no more necessary.

Remark 8: There is, as well known, a huge literature on 'classic' PIDs and PIs in order to give efficient rules for the gain tuning. Those recipes are too often rather intricate. See the two books by Åström and Hägglund (2006), O'Dwyer (2009), and the numerous references therein.

Remark 9: Output reference trajectories of the form y^* do not seem to be familiar in industrial applications of classic PIDs. This absence often leads to disturbing oscillations, and mismatches like overshoots and undershoots. Selecting y^* plays of course a key rôle in the implementation of the control synthesis. Mimicking for this tracking the highly effective feed-forward flatness-based viewpoint (see, Fliess et al., 1995; Lévine, 2009; Sira-Ramírez & Agrawal, 2004; and the numerous references in those two books) is achieved in Section 5.1 where a part of the system, which happens to be flat, is already known. This is unfortunately impossible in general: are systems like (31) and/or (32) in the appendix flat or not? Even if the above systems were flat, it might be difficult then to verify if y is a flat output or not.

Remark 10: For obtaining a suitable trajectory planning, impose to y to satisfy a given ordinary differential equation. It permits moreover if the planning turns out to be poor because of some abrupt change to replace quite easily the preceding equation by another one.

2.2 Other possible intelligent controllers

The *generalised proportional-integral* controllers, or *GPIs*, were introduced by Fliess, Marquez, Delaleau, and Sira-Ramírez (2002) in order to tackle some tricky problems like those stemming from non-minimum phase systems. Several practical case studies have confirmed their usefulness (see Morales & Sira-Ramírez, 2011; Sira-Ramírez, 2003). Although it would be possible to define their *intelligent* counterparts in general, we are limiting ourselves here to a single case which will be utilised in Section 5.6. Replace the ultra-local model (9) by

$$\dot{y} = F + \alpha u + \beta \int u, \tag{12}$$

where $\alpha, \beta \in \mathbb{R}$ are constant. Set in Equation (6)

$$\mathfrak{C}(e) = K_P e + K_I \int e + K_{II} \int \int e, \tag{13}$$

where $K_I, K_{II} \in \mathbb{R}$ are suitable constant gains. See Section 6.3 for an analogous regulator.

3. Online estimation of F

Our first publications on model-free control were proposing for the estimation of F recent techniques on the numerical differentiations of noisy signals (see Fliess, Join, & Sira-Ramírez, 2008; Liu, Gibaru, & Perruquetti, 2011; Mboup, Join, & Fliess, 2009) for estimating $y^{(\nu)}$ in Equation (1). Existing applications were until today based on a simple version of this differentiation procedure, which is quite close to what is presented in this section, namely, the utilisation of the parameter identification techniques by Fliess and Sira-Ramírez (2003, 2008).

3.1 General principles

The approximation of an integrable function, i.e., of a quite general function $[a, b] \to \mathbb{R}$, $a, b \in \mathbb{R}$, $a < b$, by a *step* function F_{approx}, i.e., a piecewise constant function, is classic in mathematical analysis (see the excellent textbooks by Godement, 1998 and Rudin, 1976). A suitable approximate estimation of F in Equation (1) boils down, therefore, to the estimation of the constant parameter ϕ in Equation (2) if it can be achieved during a sufficiently 'small' time interval. Analogous estimations of F may be carried on via the intelligent controllers (4)-(8)-(10)-(11).

3.2 Identifiability via operational calculus

3.2.1 Operational calculus

In order to encompass all the previous equations, where F is replaced by F_{approx}, consider the equation, where the classic rules of operational calculus are utilised (Mikusiński, 1983; Yosida, 1984):

$$L_1(s)Z_1 + L_2(s)Z_2 = \frac{\phi}{s} + I(s), \tag{14}$$

where

- ϕ is a constant real parameter, which has to be identified;
- $L_1, L_2 \in \mathbb{R}[s, s^{-1}]$ are Laurent polynomials;
- $I \in \mathbb{R}[s]$ is a polynomial associated to the initial conditions.

Multiplying both sides of Equation (14) by $\frac{d^N}{ds^N}$, where N is large enough, permits to get rid of the initial conditions.

It yields the *linear identifiability* (Fliess & Sira-Ramírez, 2003, 2008) of ϕ thanks to the formula:

$$\frac{(-1)^N N!}{s^{N+1}}\phi = \frac{d^N}{ds^N}\left(L_1(s)Z_1 + L_2(s)Z_2\right). \tag{15}$$

Multiplying both sides of Equation (15) by s^{-M}, where $M > 0$ is large enough, permits to get rid of positive powers of s, i.e., of derivatives with respect to time.

Remark 11: Sometimes it might be interesting in practice to replace s^{-M} by a suitable rational function of s, i.e., by a suitable element of $\mathbb{R}(s)$.

3.2.2 Time domain

The remaining negative powers of s correspond to iterated time integrals. The corresponding formulae in the time domain are easily deduced thanks to the correspondence between $\frac{d^\kappa}{ds^\kappa}$, $\kappa \geq 1$, and the multiplication by $(-t)^\kappa$ in the time domain (see some examples in Section 3.4). They may be easily implemented as discrete linear filters.

3.3 Noise attenuation

The notion of noise, which is usually described in engineering and, more generally, in applied sciences via probabilistic and statistical tools, is borrowed here from Fliess (2006) (see also Lobry & Sari, 2008 and the references therein on *nonstandard analysis*). Then the noise is related to *quick fluctuations* around zero. Such a fluctuation is a Lebesgue-integrable real-valued time function $\mathcal{F}$ which is characterised by the following property:

> its integral $\int_{\tau_i}^{\tau_f} \mathcal{F}(\tau)d\tau$ over any finite interval is *infinitesimal*, i.e., very 'small'.
> The robustness with respect to corrupting noises is thus explained thanks to Section 3.2.2.

Remark 12: This standpoint on denoising has not only been confirmed by several applications of model-free control, which were already cited in the introduction, but also by numerous ones in model-based linear control and in signal processing (see Fliess, Join, & Mboup, 2010; Gehring, Knüppel, Rudolph, & Woittennek, 2012; Morales, Nieto, Trapero, Chichamo, & Pintado, 2011; Pereira, Trapero, Muñoz, & Feliu, 2009, 2011; Trapero, Sira-Ramírez, & Battle, 2007a, 2007b, 2008). Note moreover that the nonlinear estimation techniques advocated by Fliess et al. (2008) exhibit for the same reason 'good' robustness properties, which were already illustrated by several case studies (see Menhour, d'Andréa-Novel, Boussard, Fliess, & Mounier, 2011; Morales, Feliu, & Sira-Ramírez, 2011; and the references therein).

3.4 Some more explicit calculations

3.4.1 First example

With Equation (9), Equation (14) becomes

$$sY = \frac{\phi}{s} + \alpha U + y_0,$$

where

- y_0 is the initial condition corresponding to the time interval $[t - L, t]$,
- ϕ is a constant.

Get rid of y_0 by multiplying both sides by $\frac{d}{ds}$:

$$y + s\frac{dy}{ds} = -\frac{\phi}{s^2} + \alpha\frac{du}{ds}$$

Multiplying both sides by s^{-2} for smoothing the noise yields in time domain yields

$$\phi = -\frac{6}{L^3}\int_{t-L}^{t}((L - 2\sigma)y(\sigma) + \alpha\sigma(L - \sigma)u(\sigma))\,d\sigma$$

where L is quite small.

Remark 13: L depends of course on

- the sampling period,
- the noise intensity.

Both may differ a lot as demonstrated by the numerous references on concrete case-studies given at the beginning of the introduction.

3.4.2 Second example

Close the loop with the iP (11). It yields

$$\phi = \frac{1}{L}\left[\int_{t-L}^{t}(\dot{y}^\star - \alpha u - K_P e)\,d\sigma\right].$$

4. When is the order $\nu = 1$ enough?

A most notable exception in the choice of a first order ultra-local model, i.e., $\nu = 1$ in Equation (1), is provided by the magnetic bearing studied by De Miras et al. (2012), where the friction is almost negligible. Start, therefore, with the elementary constant linear system:

$$\ddot{y} + c\dot{y} + 4y = u, \tag{16}$$

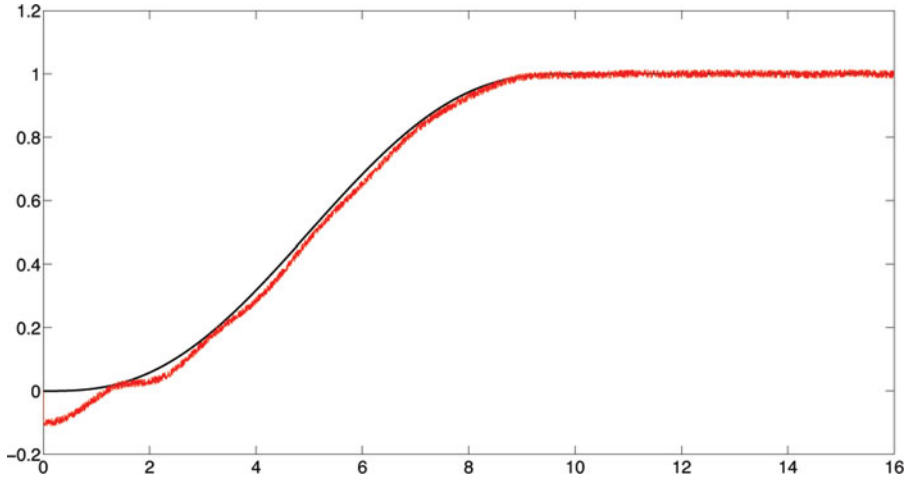


Figure 1. System output and reference.

where $c\dot{y}$ stands for some elementary friction. Figures 1 and 2 yield satisfactory numerical simulations with a iPI corrector. The following values were selected for the parameters: $c = 3, \alpha = 1, K_P = 16, K_I = 25$. With a harmonic oscillator, where $c = 0$, Figure 3 displays on the other hand a strong degradation of the performances with an iPI. Lack of friction in a given system might be related to the absence of $\dot{y}$ in the unknown equation. Taking $\nu = 1$ in Equation (1) would, therefore, yield an 'algebraic loop', which adds numerical instabilities and, therefore, deteriorates the control behaviour.

5. Numerical experiments

In the subsequent simulations, the sampling time is $T_e = 0.01$s. The corrupting noise is additive, normal, zero-mean, with a standard deviation equal in Sections 5.1 and 5.5 to 0.01, and to 0.03 elsewhere.

5.1 Control with a partially known system

5.1.1 A crude description

Consider the nonlinear Duffing spring with friction:

$$m\ddot{y} = -\mathcal{K}(y) + \mathcal{F}(\dot{y}) - d\dot{y} + F_{\text{ext}}, \tag{17}$$

where

- y is the length of the spring,
- m is a point mass,
- $F_{\text{ext}} = u$ is the control variable,
- $\mathcal{K}(y) = k_1 y + k_3 y^3$ is the resulting force from the Hooke law and the Duffing cubic term,
- $d\dot{y}$ is a classic linear friction and $\mathcal{F}(\dot{y})$ a nonlinear one. The term $\mathcal{F}(\dot{y})$ corresponds to the Tustin friction (Tustin, 1947) (see also Olsson, Aström, Canudas de Wit, Gäfvert, & Lischinsky, 1998), which is rather

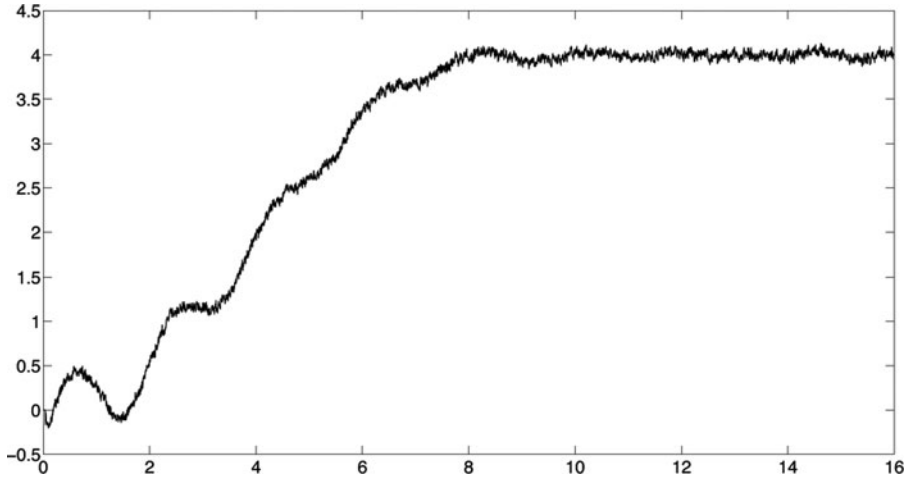


Figure 2. iPI control.

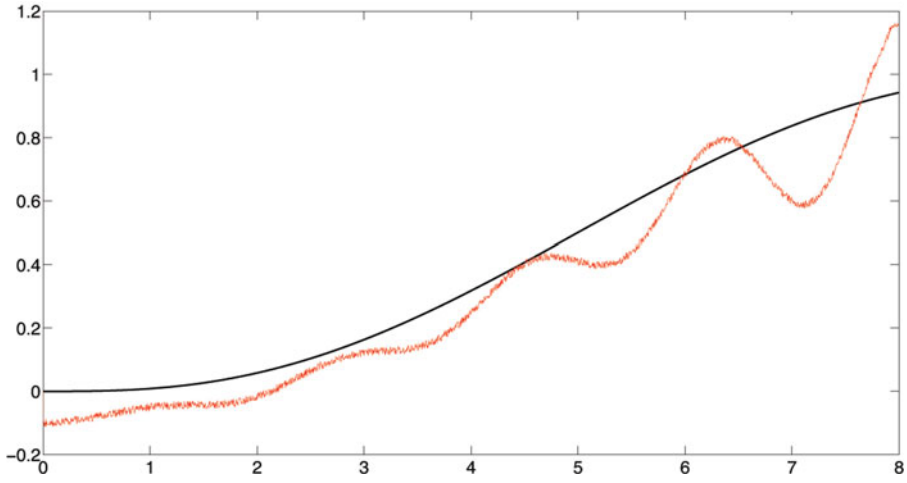


Figure 3. System output and reference.

violent with respect to the sign change of the speed (see Figure 4).

Set $m = 0.5$, $k_1 = 3$. The partially known system

$$m\ddot{y} = k_1 y + u$$

is flat, and y is a flat output. It helps us to determine a suitable reference trajectory $y^\star$ and the corresponding nominal control variable $u^\star = m\ddot{y}^\star + k_1 y^\star$. In the numerical simulations, we utilise $\hat{k}_1 = 2$, $d = 1$, $k_3 = 2$, which are in fact unknown.

5.1.2 *A PID controller*

Set $u = u^\star + v$. Associate to v a PID corrector for alleviating the tracking error $e = y - y^\star$ by imposing a denominator of the form $(s + 1.5)^3$. The corresponding tuning gains are $k_P = 1.375$, $k_I = 1.6875$, $k_D = 2.25$.

5.1.3 *iPID*

The main difference of the iPID is the following one: The presence of F which is estimated in order to compensate the nonlinearities and the perturbations like frictions. For comparison purposes, its gains are the same as previously.

5.1.4 *iP*

We do not take any advantage of Equation (17). The error tracking dynamics is again given with a pole equal to -1.5, i.e., by the denominator $(s + 1.5)$.

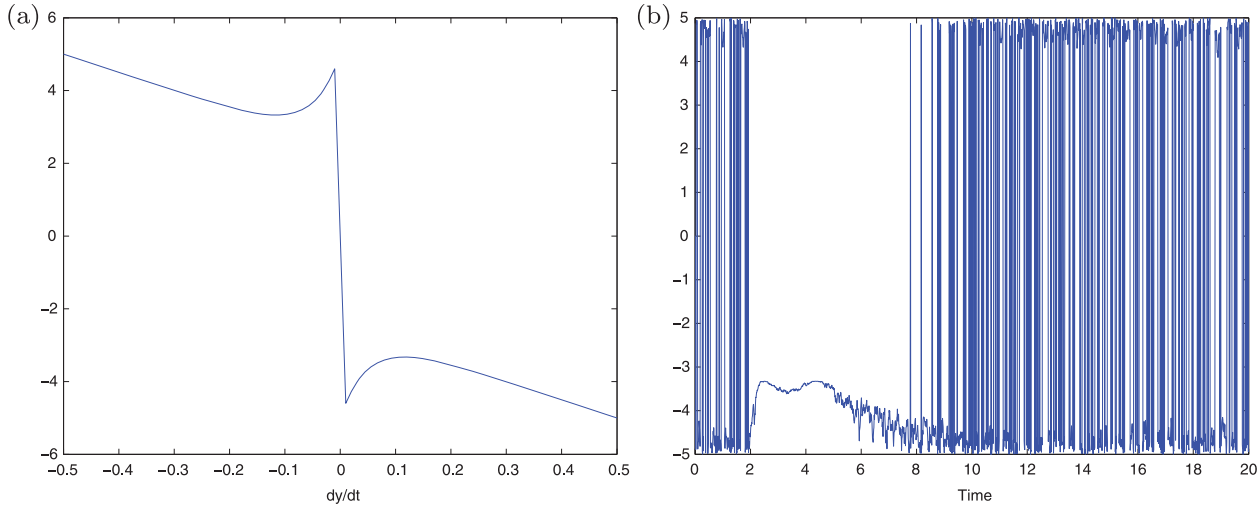


Figure 4. Model and time evolution of friction. (a) Tustin's model. (b) Friction with the iPID.

5.1.5 Numerical experiments

Figure 5 shows quite poor results with the PID of Section 5.1.2. They become excellent with the iPID and correct with iP. The practician might be right to prefer this last control synthesis

- where the implementation is immediate,
- if a most acute precision may be neglected.

Remark 14: See also Villagra et al. (2009) and Villagra and Balaguer (2011) for concrete examples related to guided vehicles.

5.2 Robustness with respect to system's changes

The examples below demonstrate that if the system is changing, our intelligent controllers behave quite well without the need of any new calibration.

5.2.1 Scenario 1: the nominal case

The nominal system is defined by the transfer function

$$\frac{(s+2)^2}{(s+1)^3}. \tag{18}$$

A tuning of a classic PID controller

$$u = k_p e + k_i \int e + k_d \dot{e}, \tag{19}$$

where

- $e = y - y^*$ is the tracking error,
- $k_p, k_i, k_d \in \mathbb{R}$ are the gains,

yields via standard techniques (see Åström & Hägglund, 2006) $k_p = 1.8177$, $k_i = 0.7755$, $k_d = 0.1766$. A low-pass filter is moreover added to the derivative $\dot{e}$ in order to attenuate the corrupting noise. Our model-free approach utilises the ultra-local model $\dot{y} = F + u$ and an iP (11), where $K_P = 1.8177$. Figure 6 shows perhaps a slightly better behaviour of the iP.

5.2.2 Scenario 2: modifying the pole

A system change, aging for instance, might be seen by as new pole -2.2 in the transfer function (18) which becomes

$$\frac{(s+2)^2}{(s+2.2)^3}.$$

As shown in Figure 7, without any new calibration the performances of the PID worsen whereas those of the iP remain excellent.

5.2.3 Scenario 3: actuator's fault

A power loss of the actuator occurs at time $t = 8$s. It is simulated by dividing the control by 2 at $t = 8$s. Figure 8 shows an accommodation of the iP which is much faster than with the PID.

Remark 15: Sections 5.2.2 and 5.2.3 may be understood as instances of *fault accommodation*, which contrarily to most of the existing literature are not model-based (see also Moussa Ali, Join, & Hamelin, 2012). It is perhaps worth mentioning here that model-based fault diagnosis has also benefited from the estimation techniques summarised in Section 3 (see Fliess, Join, & Sira-Ramírez, 2004; Fliess et al., 2008).

5.3 A nonlinear system

Take the following academic unstable nonlinear system

$$\dot{y} - y = u^3.$$

The clssic PID (19) is tuned with $k_p = 2.2727$, $k_i = 1.8769$, $k_d = 0.1750$. The simulations depicted in Figure 9 shows a poor trajectory tracking for small values of the reference trajectory. The iP, which is related to the ultra-local model $\dot{y} = F + u$, corresponds to $K_P = 2.2727$. Its excellent performances in the whole operating domain are also shown in Figure 9.

5.4 Delay systems

Consider the system

$$\dot{y}(t) = y(t) + 5y(t-\tau) + u,$$

where moreover the delay τ, $0 \leq \tau \leq 5$s is not assumed to be

- known,
- constant.

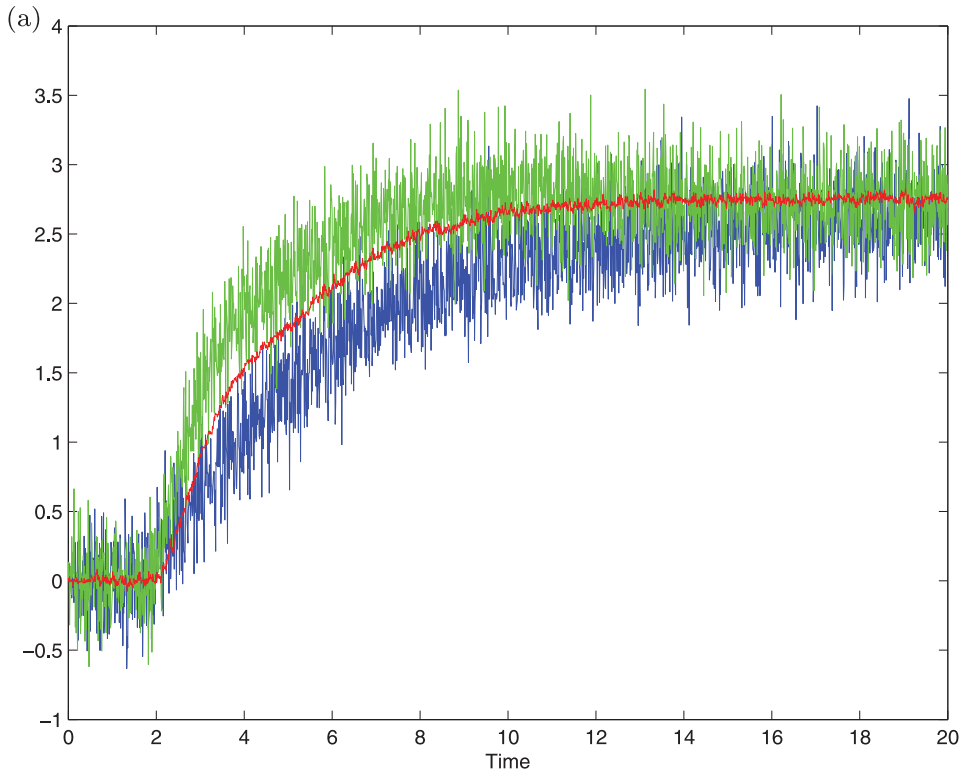
Set for the numerical simulations (see Figure 10)

$$\tau(t) = \tau(t - T_e) + 10T_e \text{sign}(N(t)), \quad \tau(0) = 2.5\text{s},$$

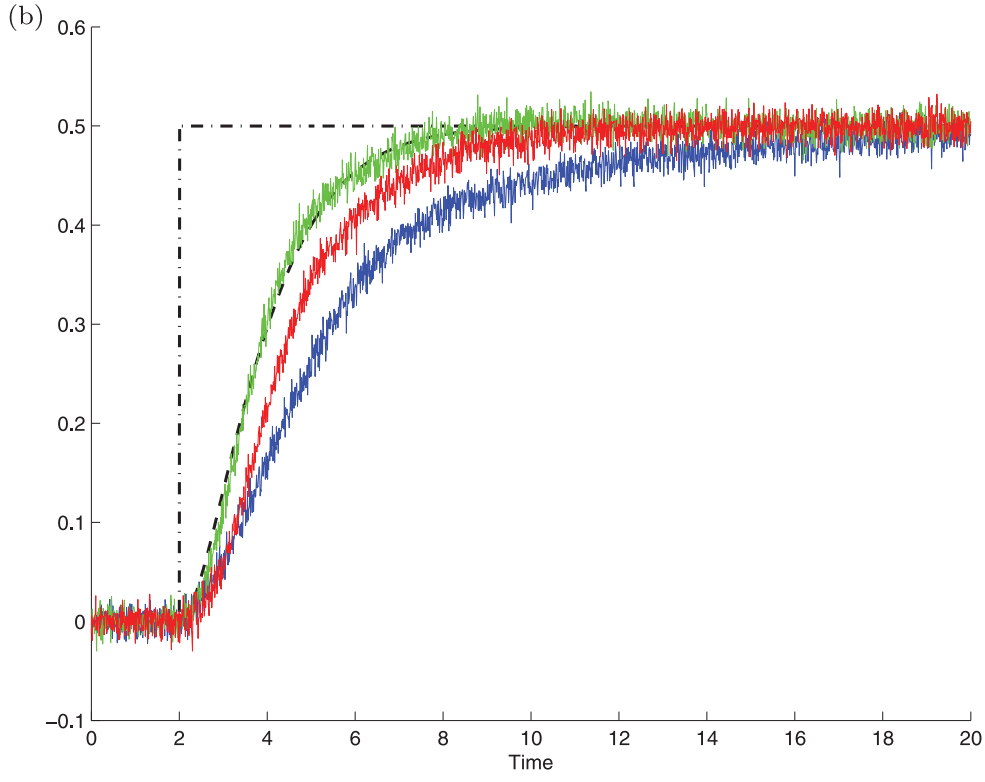
where N is a zero-mean Gaussian distribution with standard deviation 1. An iP where $K_P = 1$ is deduced from the ultra-local model $\dot{y} = F + u$. The results depicted in Figure 11 are quite satisfactory.

Remark 16: Extending the above control strategy to nonlinear systems and to neutral systems is straightforward. It will not be developed here.

Remark 17: The delay appearing with the hydro-electric power plants studied by Join et al. (2010b) was taken into

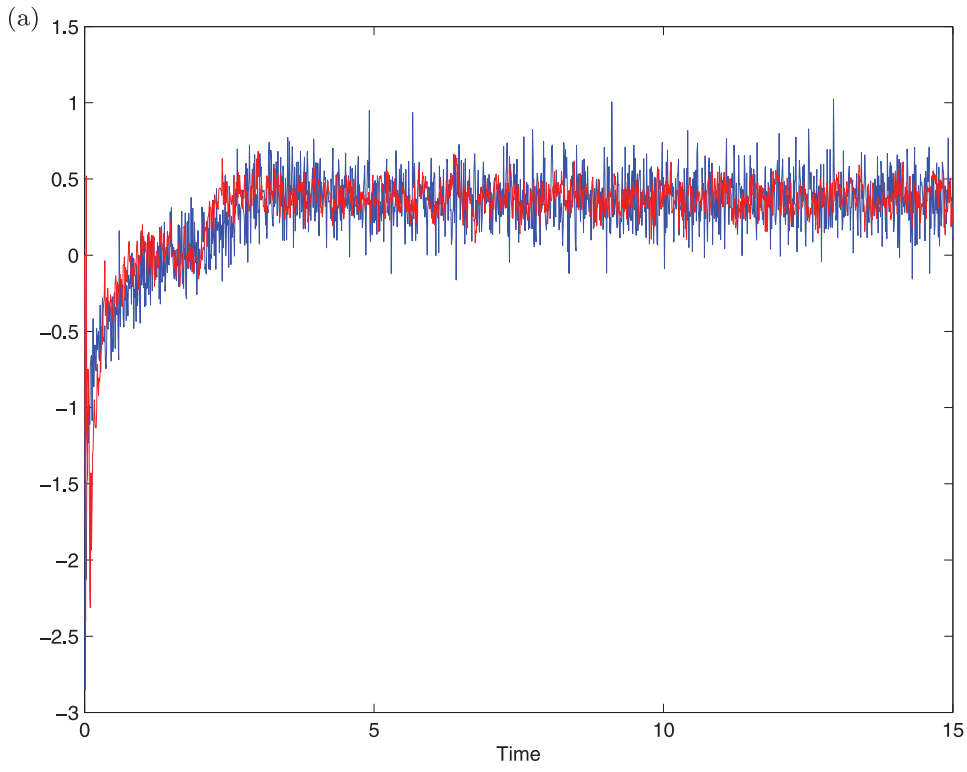


Controls $F_{\text{ext}}(t)$: PID(–, blue), iP(–, red), iPID(–, green)

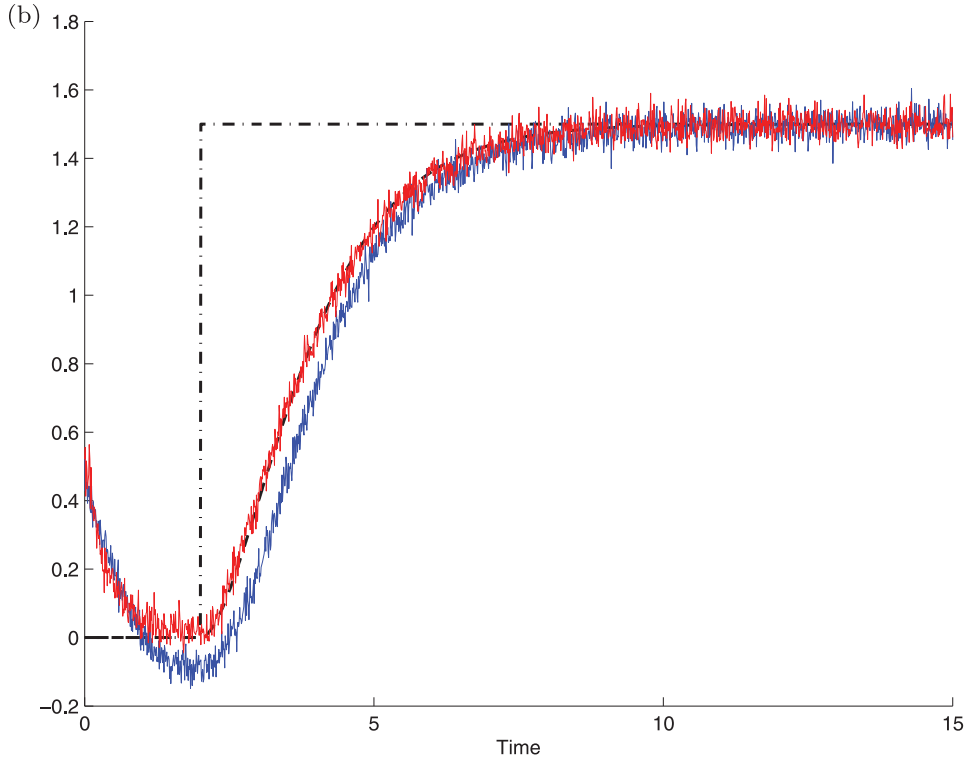


Setpoint (- ., black), reference (- -, black), and output: PID(–, blue), iP(–, red), iPID(–, green)

Figure 5. Partially known system: comparisons. (a) Controls $F_{\text{ext}}(t)$: PID(–, blue), iP(–, red), iPID(–, green). (b) Setpoint (- ., black), reference (- -, black), and output: PID(–, blue), iP(–, red), iPID(–, green). Colour visible online.

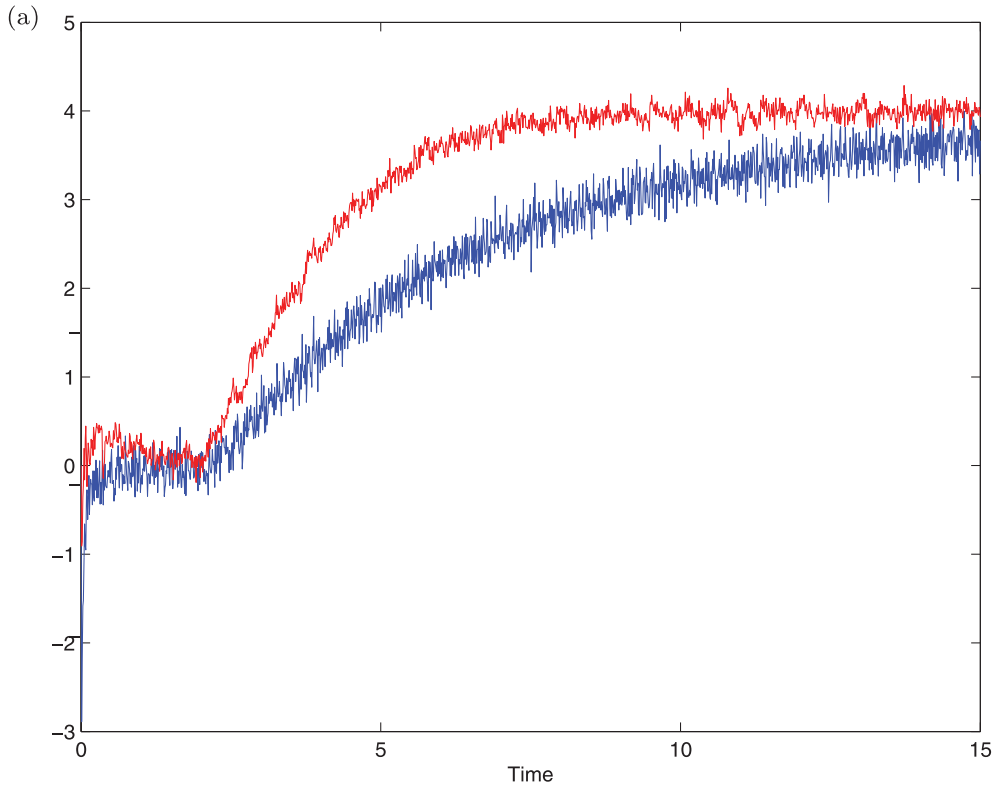


Controls: PID(–, blue), iP(–, red)

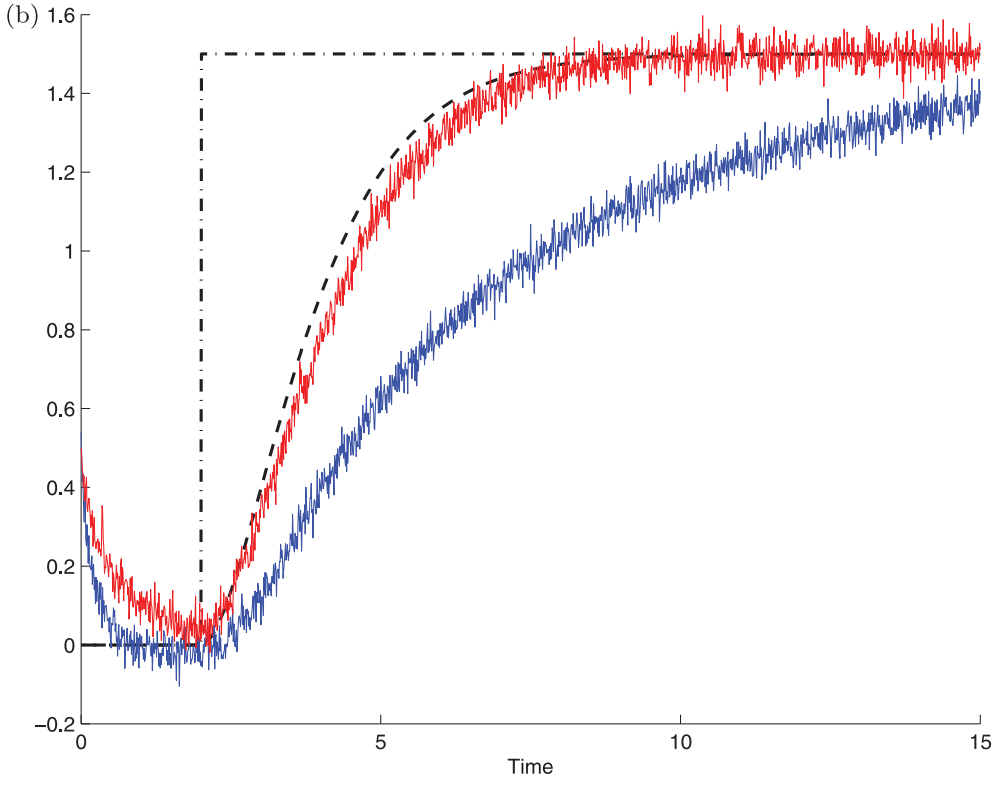


Setpoint (- ., black), reference (- -, black), and outputs: PID(–, blue), iP(–, red)

Figure 6. Scenario 1: comparisons. (a) Controls: PID(–, blue), iP(–, red). (b) Setpoint (- ., black), reference (- -, black), and outputs: PID(–, blue), iP(–, red). Colour visible online.



Controls: PID(–, blue), iP(–, red)



Setpoint (- ., black), reference (- -, black), and outputs: PID(–, blue), iP(–, red)

Figure 7. Scenario 2: comparisons. (a) Controls: PID(–, blue), iP(–, red). (b) Setpoint (- ., black), reference (- -, black), and outputs: PID(–, blue), iP(–, red). Colour visible online.

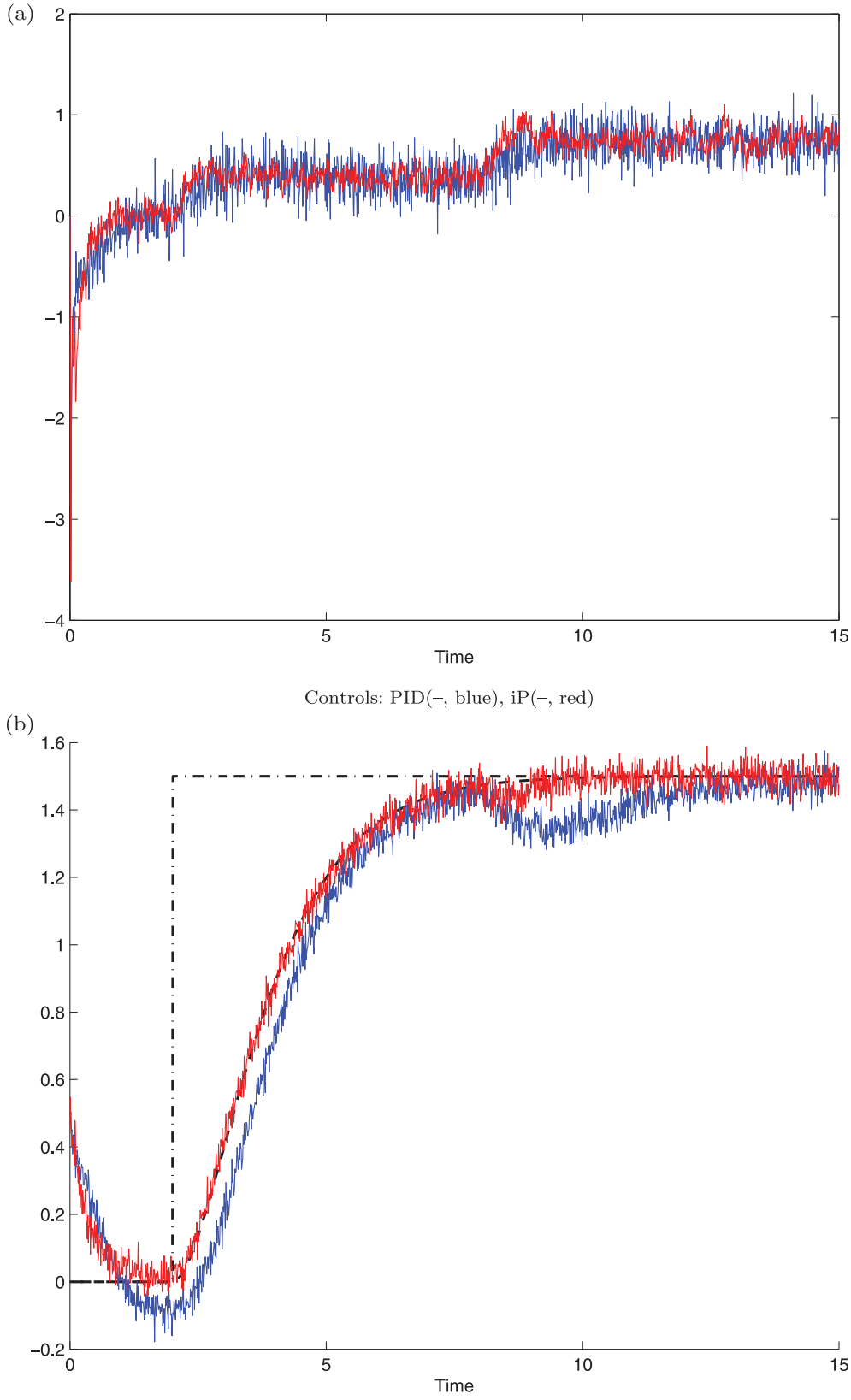
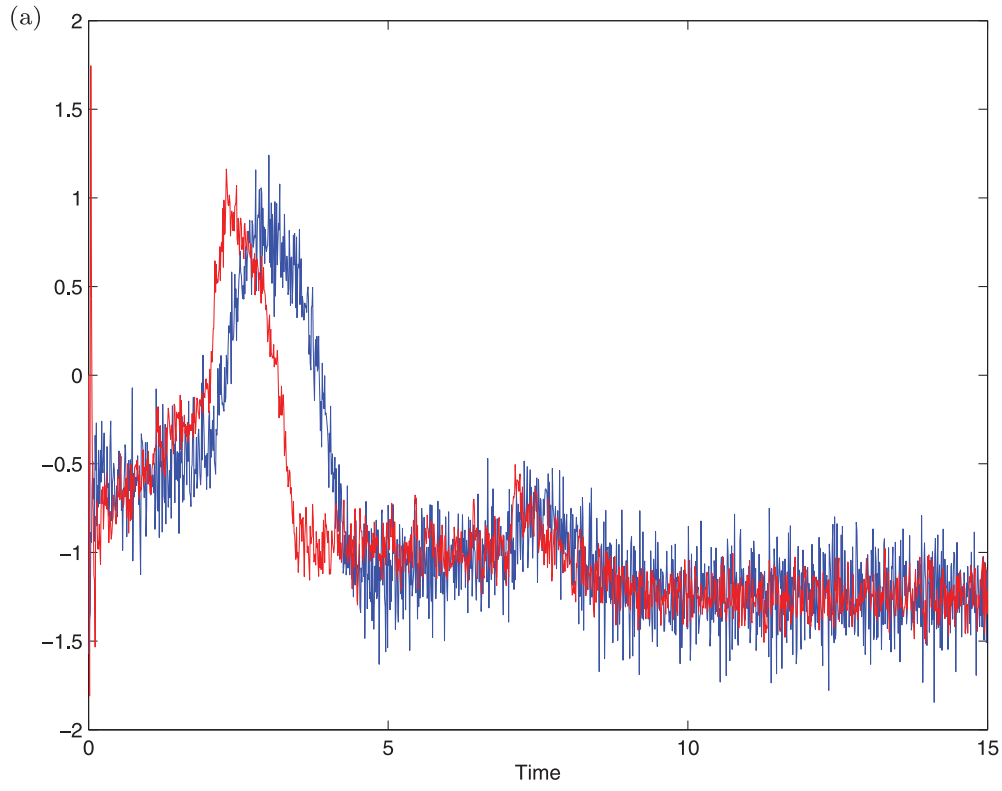
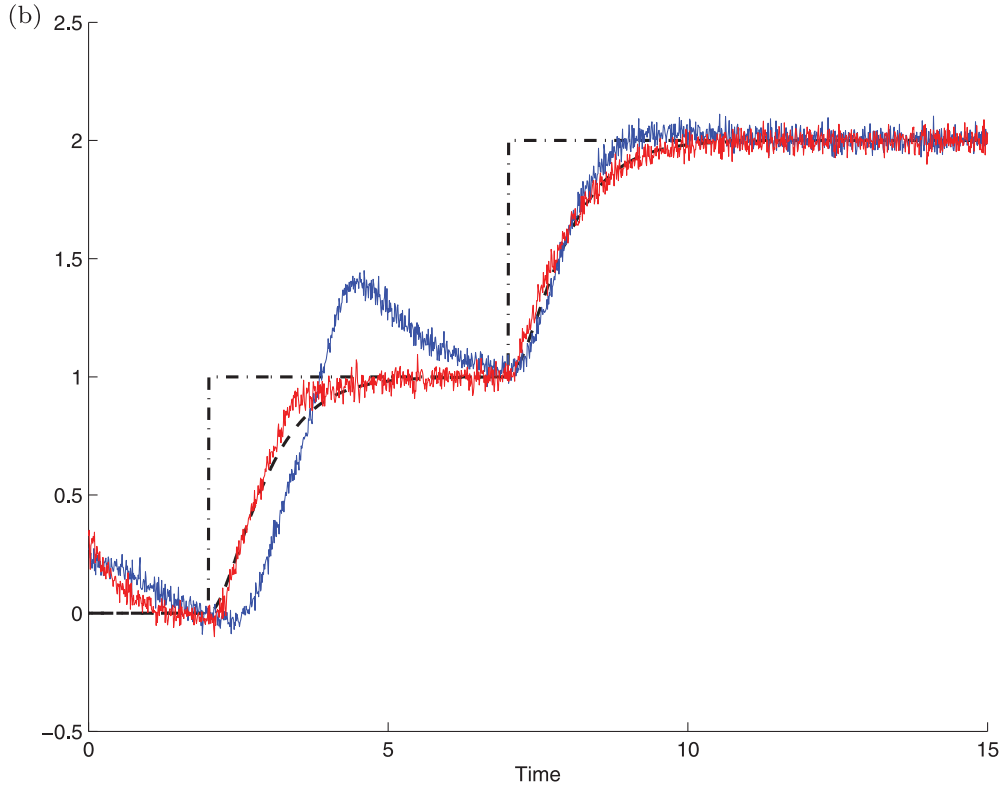


Figure 8. Scenario 3: comparisons. (a) Controls: PID(–, blue), iP(–, red). (b) Setpoint (- ., black), reference (- -, black), and outputs: PID(–, blue), iP(–, red). Colour visible online.



Controls: PID(–, blue), iP(–, red)



Setpoint (- ., black), reference (- -, black), and outputs: PID(–, blue), iP(–, red)

Figure 9. Nonlinear system: comparisons. (a) Controls: PID(–, blue), iP(–, red). (b) Setpoint (- ., black), reference (- -, black), and outputs: PID(–, blue), iP(–, red). Colour visible online.

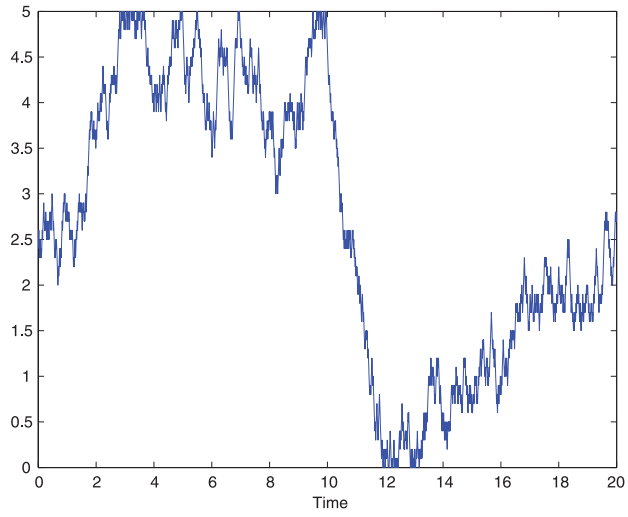


Figure 10. Delay function.

account via an empirical knowledge of the process. Some numerical tabulations were employed in order to get in some sense 'rid' of the delay. Such a viewpoint might be the most realistic one in industry.

Remark 18: We only refer here to 'physical' delays and not to the familiar approximation in engineering of 'complex' systems via delays ones (see Shinskey, 1996). Let us emphasise that this type of approximation is loosing its importance in our setting.

5.5 A one-dimensional semi-linear heat equation

The heat equation is certainly one of the most studied topic in mathematical physics. It would be pointless to review its corresponding huge bibliography even in the control domain, where many of the existing high-level control theories have been tested. Consider with Coron and Trélat (2004) the one-dimensional semi-linear heat equation:

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial x^2} + f(w), \tag{20}$$

where

- $0 \leq x \leq L$,
- $w(t, 0) = c$,
- $w(t, L) = u(t)$ is the control variable,
- $w(0, x) = sin(\pi x) + (u(0) - c)x + c$, where $c \in \mathbb{R}$ is a constant.

We want to obtain given time-dependent temperature at $x = x_c$. Consider the following scenarios:

(1) $x_c = 1/3L, f = 0, c = 0$,
(2) $x_c = 1/3L, f = 0, c = 0.5$,
(3) $x_c = 2/3L, f = 0, c = 0$,
(4) $x_c = 2/3L, f = y^3, c = 0$.

The control synthesis is achieved thanks to the elementary one-dimensional ultra-local model

$$\dot{y} = F + 10u$$

and the straightforward iP, where $K_p = 10$. The four numerical simulations, displayed by Figures 12, 13, 14, and 15, are quite convincing.

5.6 A peculiar non-minimum phase system

Consider the non-minimum phase system defined by the transfer function:

$$\frac{(s-1)}{(s+1)(s+2)}. \tag{21}$$

Utilise Equations (12) and (13). Set $\alpha = -\beta = 10$, $K_P = 3$ and $K_I = K_{II} = 5$. Figure 16 displays good performances.

Remark 19: It is easy to check that the above calculations work only for a single unstable zero, like in Equation (21). Our approach cannot be extended to arbitrary non-minimum phase systems.

Remark 20: It is well known that the control synthesis of a non-minimum phase system is even a difficult task with a perfectly known mathematical model. Among the many solutions which have been suggested in the literature, let us mention a flatness-based output change (see Fliess & Marquez, 2000; Fliess et al., 1998). When a mathematical model is unknown or poorly known, the non-minimum phase character of an output cannot be deduced mathematically but only via a 'bad' qualitative behaviour of this output. Selecting a minimum phase output, i.e., an output with 'good' qualitative properties, might be a more realistic alternative. It necessitates nevertheless an excellent 'practical' knowledge of the plant behaviour.

6. Connections between classic and intelligent controllers

The results below connect classic PIDs to our intelligent controllers. They explain, therefore, why classic PIDs are used in rather arbitrary industrial situations thanks to a fine gain tuning, which might be quite difficult to achieve in practice.

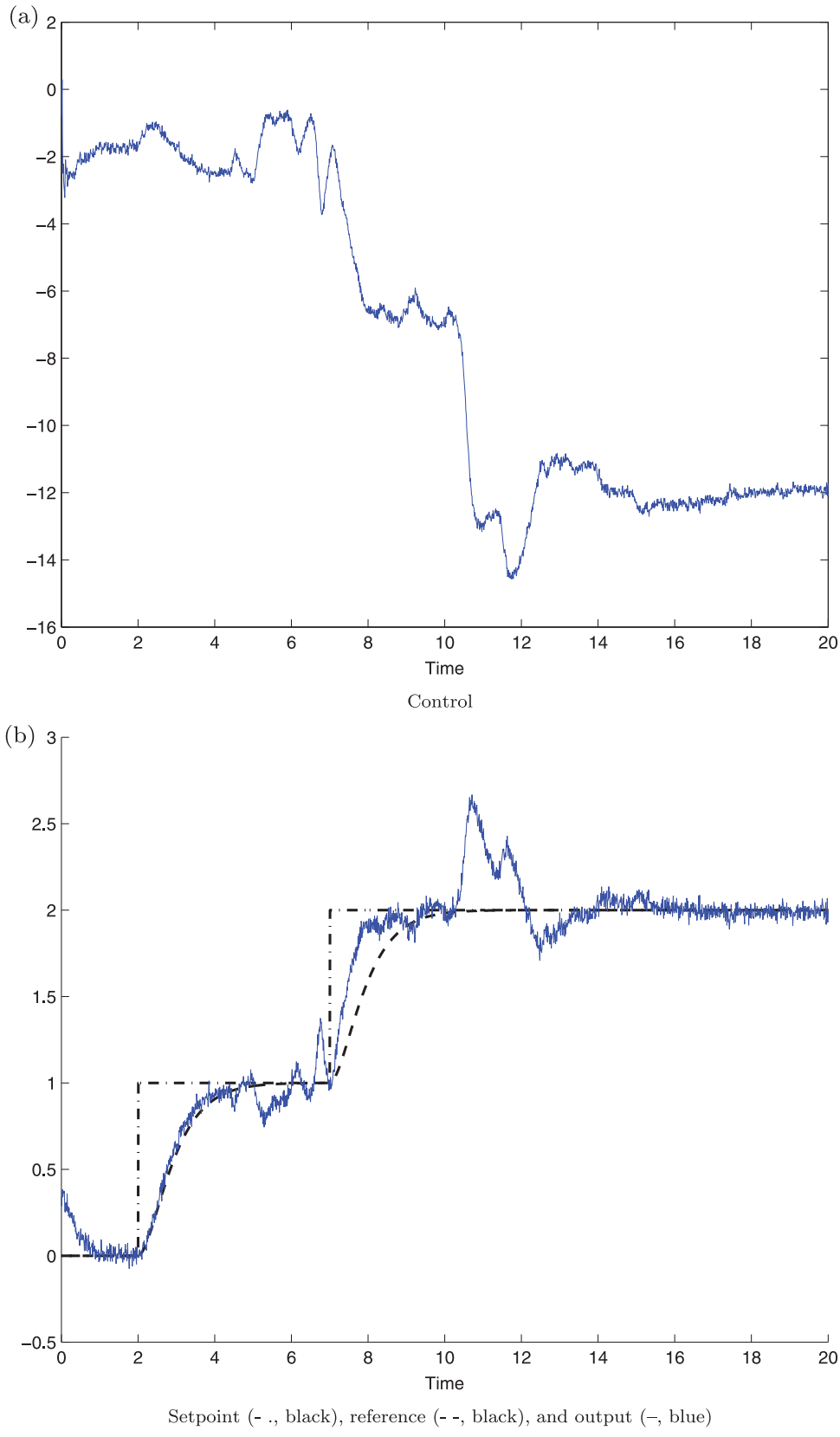


Figure 11. Delay system: Model-free control. (a) Control. (b) Setpoint (- ., black), reference (- -, black), and output (–, blue).

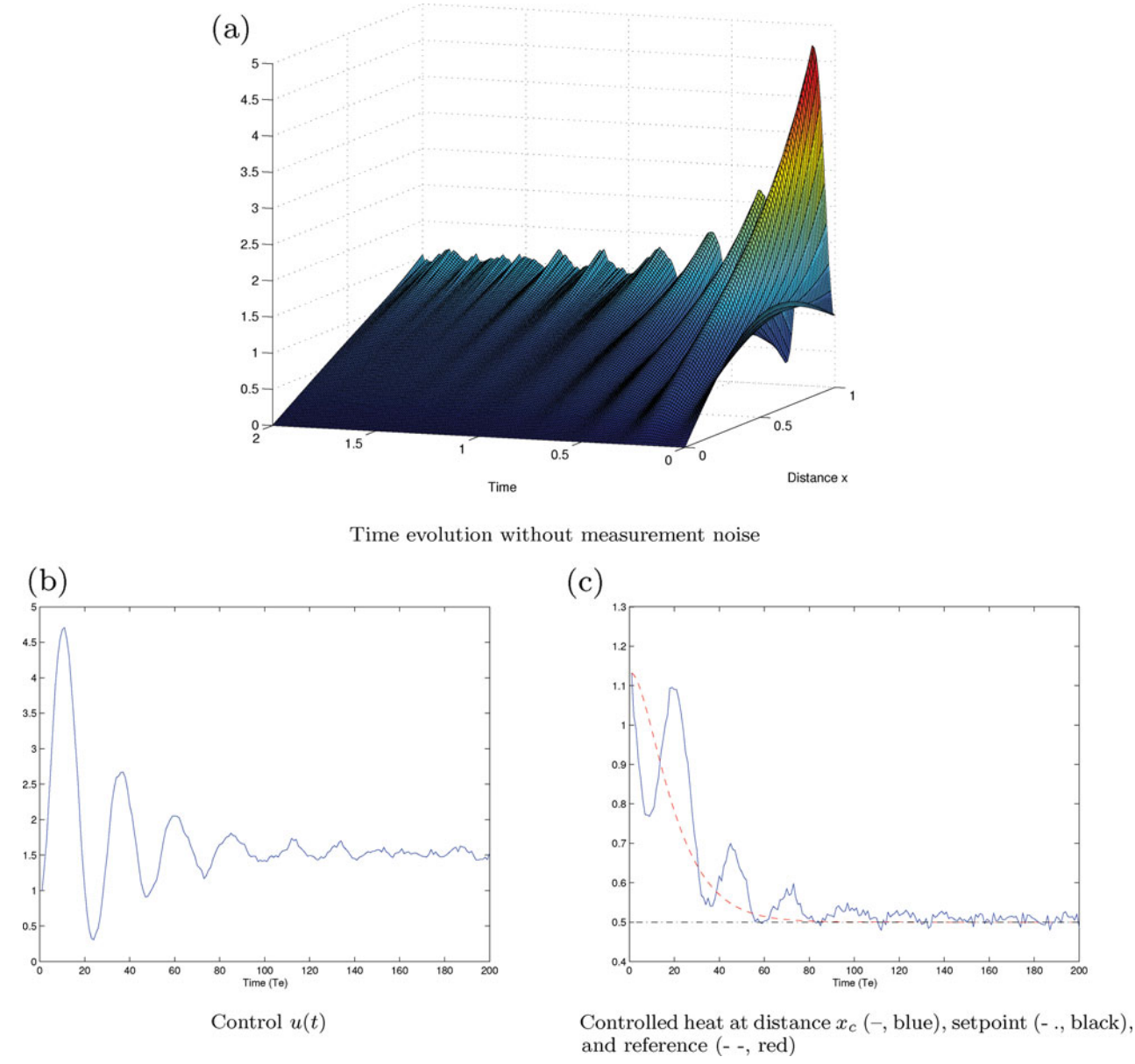


Figure 12. Heat equation: scenario 1. (a) Time evolution without measurement noise. (b) Control $u(t)$. (c) Controlled heat at distance x_c (–, blue), setpoint (- ., black), and reference (- -, red). Colour visible online.

6.1 PI and iP

6.1.1 A crude sampling of PIs

Consider the classic continuous-time PI controller:

$$u(t) = k_p e(t) + k_i \int e(\tau) d\tau. \tag{22}$$

A crude sampling of the integral $\int e(\tau)d\tau$ through a Riemann sum $I(t)$ leads to

$$\int e(\tau)d\tau \simeq I(t) = I(t-h) + he(t),$$

where h is the sampling interval. The corresponding discrete form of Equation (22) reads

$$u(t) = k_p e(t) + k_i I(t) = k_p e(t) + k_i I(t-h) + k_i h e(t).$$

Combining the above equation with

$$u(t-h) = k_p e(t-h) + k_i I(t-h)$$

yields

$$u(t) = u(t-h) + k_p\,(e(t) - e(t-h)) + k_i h e(t). \tag{23}$$

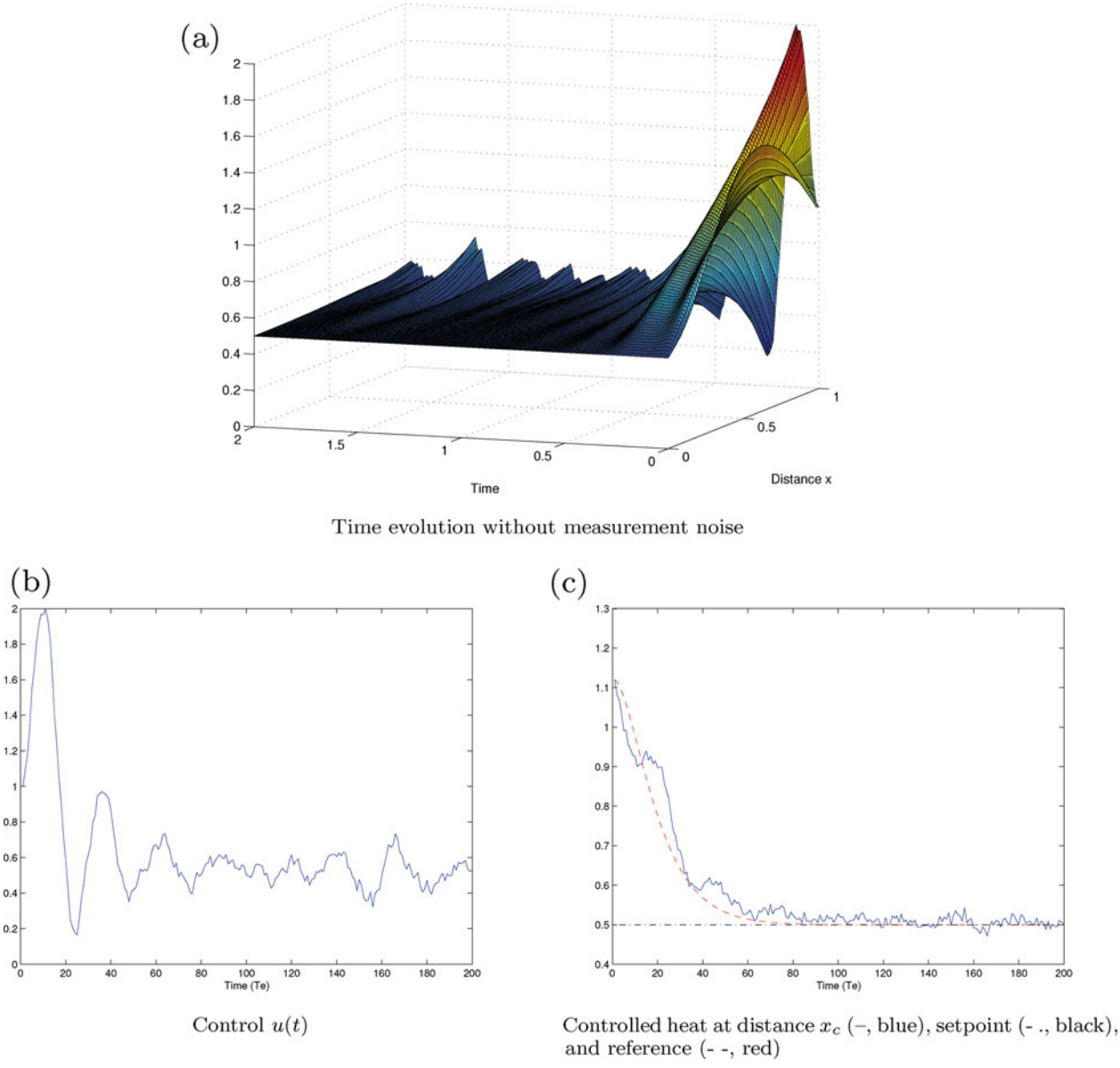


Figure 13. Heat equation: scenario 2. (a) Time evolution without measurement noise. (b) Control $u(t)$. (c) Controlled heat at distance x_c (–, blue), setpoint (- ., black), and reference (- -, red). Colour visible online.

Remark 21: A trivial sampling of the 'velocity form' of Equation (22)

$$\dot{u}(t) = k_p \dot{e}(t) + k_i e(t)$$

yields

$$\frac{u(t) - u(t-h)}{h} = k_p \left(\frac{e(t) - e(t-h)}{h} \right) + k_i e(t),$$

which is equivalent to Equation (23).

6.1.2 Sampling iPs

Utilise, if $\nu = 1$, the iP, which may be rewritten as

$$u(t) = \frac{\dot{y}^*(t) - F + K_P e(t)}{\alpha}.$$

Replace F by $\dot{y}(t) - \alpha u(t-h)$ and therefore by

$$\frac{y(t) - y(t-h)}{h} - \alpha u(t-h),$$

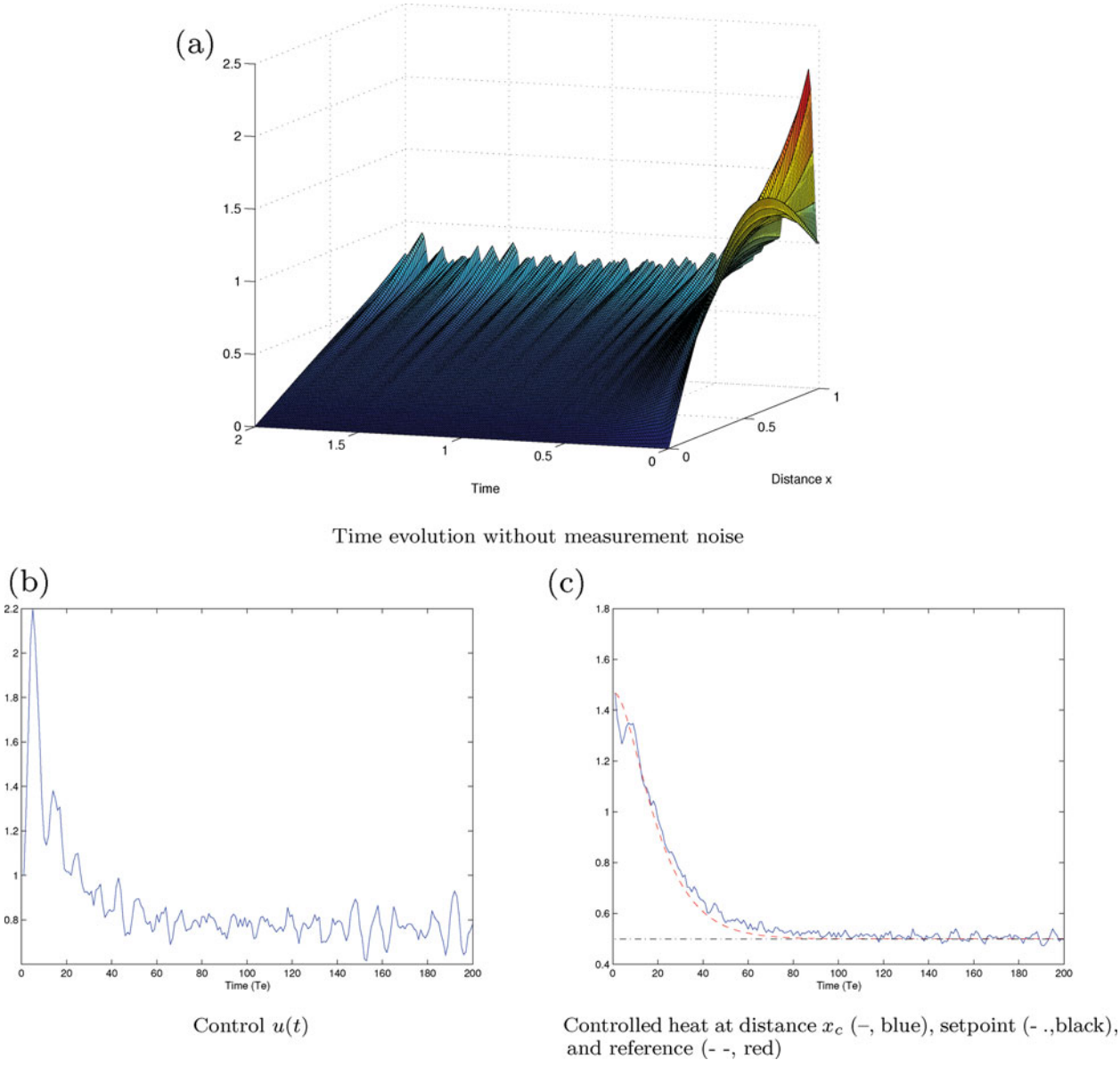


Figure 14. Heat equation: scenario 3. (a) Time evolution without measurement noise. (b) Control $u(t)$. (c) Controlled heat at distance x_c (–, blue), setpoint (- ., black), and reference (- -, red). Colour visible online.

it yields

$$u(t) = u(t-h) - \frac{e(t) - e(t-h)}{h\alpha} + \frac{K_P}{\alpha}\, e(t). \qquad (24)$$

6.1.3 Comparison

FACT. Equations (23) and (24) become **identical** if we set

$$k_p = -\tfrac{1}{\alpha h}, \quad k_i = \tfrac{K_P}{\alpha h}. \qquad (25)$$

Remark 22: It should be emphasised that the above property, defined by Equations (25), does not hold for continuous-time PIs and iPs. This equivalence is strictly related to time sampling, i.e., to computer implementation, as demonstrated by taking $h \downarrow 0$ in Equations (25).

6.2 PID and iPD

Extending the calculations of Section 6.1 is quite obvious. The velocity form of the PID

$$u(t) = k_p e(t) + k_i \int e(\tau) d\tau + k_d \dot{e}$$

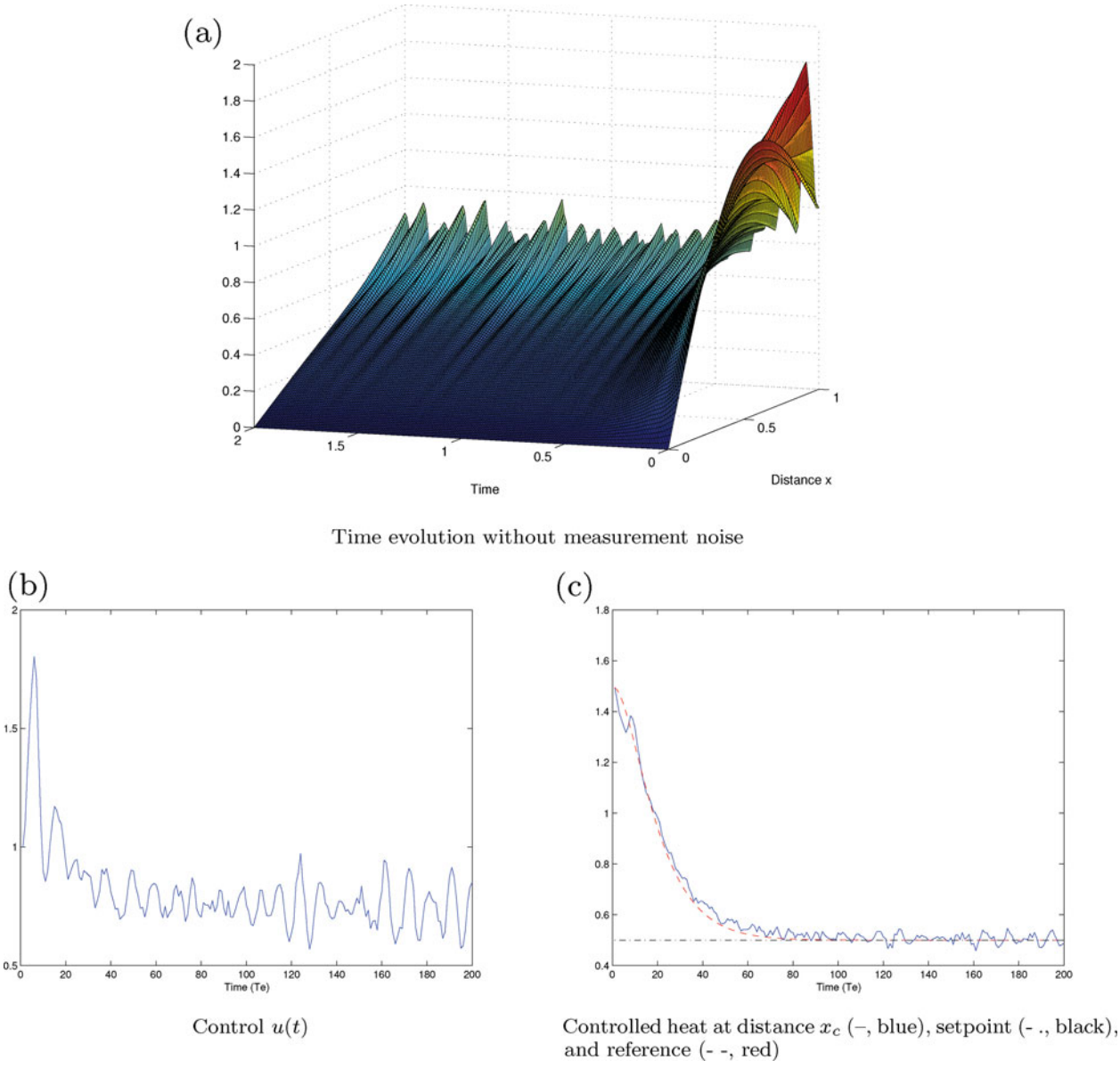


Figure 15. Heat equation: scenario 4. (a) Time evolution without measurement noise. (b) Control $u(t)$. (c) Controlled heat at distance x_c (–, blue), setpoint (- ., black), and reference (- -, red). Colour visible online.

reads $\dot{u}(t) = k_p\dot{e}(t) + k_i e(t) + k_d\ddot{e}(t)$. It yields the obvious sampling

$$u(t) = u(t-h) + k_p h\dot{e}(t) + k_i h e(t) + k_d h\ddot{e}(t) \qquad (26)$$

$\nu = 2$ on the other hand, Equation yields $u(t) = \frac{1}{\alpha}(\ddot{y}^*(t) - F + K_P e(t) + K_D\dot{e}(t))$. From the computer implementation $F = \ddot{y}(t) - \alpha u(t-h)$, we derive

$$u(t) = u(t-h) - \frac{1}{\alpha}\ddot{e}(t) + \frac{K_P}{\alpha}e(t) + \frac{K_D}{\alpha}\dot{e}(t). \qquad (27)$$

FACT. Equations (26) and (27) become **identical** if we set

$$k_p = \frac{K_D}{\alpha h}, \quad k_i = \frac{K_P}{\alpha h}, \quad k_d = -\frac{1}{\alpha h}. \qquad (28)$$

6.3 iPI and iPID

Equation (27) becomes with the iPID

$$u(t) = u(t-h) - \frac{1}{\alpha}\ddot{e}(t) + \frac{K_P}{\alpha}e(t) + \frac{K_I}{\alpha}\int e + \frac{K_D}{\alpha}\dot{e}(t). \qquad (29)$$

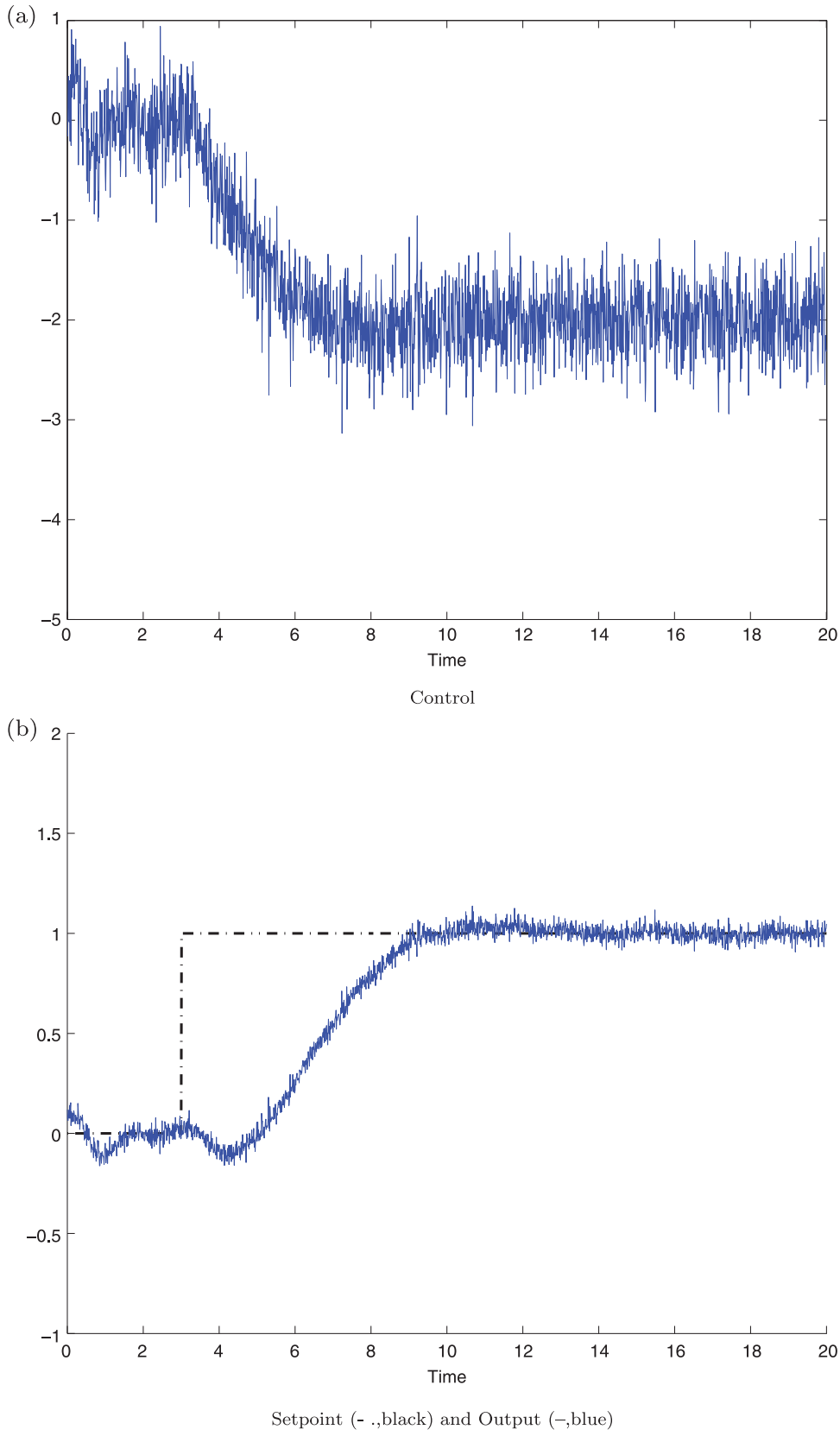


Figure 16. Non-minimum phase linear system: Model-free control. (a) Control. (b) Setpoint (- ., black) and Output (–, blue).

Table 1. Correspondence between the gains of sampled classic and intelligent controllers.

		iP	iPD	iPI	iPID
PI	k_p	$-1/\alpha h$			
	k_i	$K_P/\alpha h$			
PID	k_p		$K_D/\alpha h$		
	k_i		$K_P/\alpha h$		
	k_d		$-1/\alpha h$		
PI^2	k_p			$-1/\alpha h$	
	k_i			$K_P/\alpha h$	
	k_{ii}			$K_I/\alpha h$	
PI^{2}D	k_p				$K_D/\alpha h$
	k_i				$K_P/\alpha h$
	k_{ii}				$K_I/\alpha h$
	k_d				$-1/\alpha h$

Introduce the PI^{2}D controller

$$u(t) = k_p e(t) + k_i \int e(\tau) d\tau + k_{ii} \iint e d\tau d\sigma + k_d \dot{e}(t).$$

The double integral, which appears there, seems to be quite uncommon in control engineering. To its velocity form $\dot{u}(t) = k_p \dot{e}(t) + k_i e + k_{ii} \int e d\tau + k_d \ddot{e}(t)$ corresponds the sampling

$$u(t) = u(t-h) + k_p h \dot{e}(t) + k_i h e + k_{ii} h \int e d\tau + k_d h \ddot{e}(t),$$

which is identical to Equation (29) if one sets

$$k_p = \frac{K_D}{\alpha h}, \quad k_i = \frac{K_P}{\alpha h}, \quad k_{ii} = \frac{K_I}{\alpha h}, \quad k_d = -\frac{1}{\alpha h}. \quad (30)$$

The connection between iPIs and PI^{2}s follows at once.

6.4 *Table of correspondence*

The previous calculations yield the following correspondence (Table 1) between the gains of our various controllers.

Remark 23: Due to the form of Equation (22), it should be noticed that the tuning gains of the classic regulators ought to be negative.

7. Conclusion

Several theoretical questions remain of course open. Let us mention some of them, which appear today to be most important:

- The fact that multi-variable systems were not studied here is due to a lack until now of concrete case studies. They should, therefore, be examined more closely.
- Even if some delay and/or non-minimum phase examples were already successfully treated (see Sections 5.4 and 5.6; Andary et al., 2012; Join et al., 2010b; Riachy, Fliess, Join, & Barbot, 2010), a general understanding is still missing, like, to the best of our knowledge, with any other recent setting (see Åström & Hägglund, 2006; O'Dwyer, 2009; Xu et al., 2012). We believe as advocated in Remarks **17** and 20 that
  - looking for a purely mathematical solution might be misleading,
  - taking advantage on the other hand of a 'good' empirical understanding of the plant might lead to a more realistic track.

It goes without saying that comparisons with existing approaches should be further explored. It has already been done with

- classic PIDs here, and by Gédouin et al. (2011), Milanes et al. (2012) for some active spring and vehicles,
- some aspects of *sliding modes* by Riachy, Fliess, and Join (2011),
- *fuzzy control* for some vehicles by Milanes et al. (2012) and Villagra and Balaguer (2011).

Those comparisons were until now always favourable to our setting.

If model-free control and the corresponding intelligent controllers are further reinforced, especially by numerous fruitful applications, the consequences on the future development and teaching (see the excellent textbook by Åström & Murray, 2008) of control theory might be dramatic:

- Questions on the structure and on the parameter identification of linear and nonlinear systems might loose their importance if the need of a 'good' mathematical modelling is diminishing.
- Many effort on robustness issues with respect to a 'poor' modelling and/or to disturbances may be viewed as obsolete and therefore less important. As a matter of fact those issues disappear to a large extent thanks to the continuously updated numerical values of F in Equation (1).

Another question, which was already raised by Abouaïssa et al. (2012), should be emphasised. Our model-free control strategy yields a straightforward regulation of industrial plants, whereas the corresponding digital simulations need a reasonably accurate mathematical model in order to feed the computers. Advanced parameter identification and numerical analysis techniques might then be necessary tools (see Abouaïssa et al., 2012; Join

et al., 2010b). This dichotomy between elementary control implementations and intricate computer simulations seems to the best of our knowledge to have been ignored until today. It should certainly be further dissected as a fundamental epistemological matter in engineering and, perhaps also, in other fields.

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Appendix A. An approximation property

A.1 Functionals

We restrict ourselves to a SISO system, i.e., to a system with a single control variable u and a single output y. Even without knowing any 'good' mathematical model we may assume that the system corresponds to a *causal*, or *non-anticipative*, *functional*, i.e., for any time instant $t > 0$,

$$y(t) = \mathcal{F}\left(u(\tau)|0 \leq \tau \leq t\right), \tag{A1}$$

where $\mathcal{F}$ depends on

- the past and the present, and not on the future,
- various perturbations,
- initial conditions at $t = 0$.

Example A.1: A popular representation of rather arbitrary nonlinear systems in engineering is provided by Volterra series (see Barrett, 1963; Rugh, 1981; Lamnabhi-Lagarrigue, 1995). Such a series may be defined by

$$\begin{aligned} y(t) = h_0(t) &+ \int_0^t h_1(t,\tau)u(\tau)d\tau \\ &+ \int_0^t \int_0^t h_2(t,\tau_2,\tau_1)u(\tau_2)u(\tau_1)d\tau_2 d\tau_1 + \dots \\ &\int_0^t \dots \int_0^t h_v(t,\tau_v,\dots\tau_1)u(\tau_v)\dots u(\tau_1)d\tau_v \dots d\tau_1 \\ &+ \dots \end{aligned}$$

Solutions of quite arbitrary differential equations may be expressed as Volterra series.

A.2 The Stone-Weierstraß theorem

Let

- $\mathcal{I} \subset [0, +\infty[$ be a compact subset,
- $\mathcal{C} \subset C^0(\mathcal{I})$ be a compact subset, where $C^0(\mathcal{I})$ is the space of continuous functions $\mathcal{I} \to \mathbb{R}$, which is equipped with the topology of uniform convergence.

Consider the Banach $\mathbb{R}$-algebra $\mathfrak{S}$ of continuous causal functionals (31) $\mathcal{I} \times \mathcal{C} \to \mathbb{R}$. If a subalgebra contains a non-zero constant element and separates points in $\mathcal{I} \times \mathcal{C}$, then it is dense in $\mathfrak{S}$ according to the Stone-Weierstraß theorem (see the excellent textbooks by Choquet, 2000 and Rudin, 1967).

A.3 Algebraic differential equations

Let $\mathfrak{A} \subset \mathfrak{S}$ be the set of functionals which satisfy an algebraic differential equation of the form

$$E(y, \dot{y}, \ldots, y^{(a)}, u, \dot{u}, \ldots, u^{(b)}) = 0, \tag{A2}$$

where E is a polynomial function of its arguments with real coefficients. Satisfying Equation (A2) is equivalent saying that y is *differential algebraic* over the *differential field* $\mathbb{R}\langle u\rangle$.

Remark A1: Remind that a *differential field* (see the two following books by Chambert-Loir, 2005 and Kolchin, 1973, and the papers by Delaleau, 2002, Fliess et al., 2008 and Fliess et al., 1995) is a commutative field which is equipped with a derivation. A typical element of $\mathbb{R}\langle u\rangle$ is a rational function of $u, \dot{u}, ..., u^{(\nu)}, ...$, with real coefficients.

It is known (Kolchin, 1973) that the sum and the product of two elements which are differentially algebraic over $\mathbb{R}\langle u\rangle$ is again differentially algebraic over $\mathbb{R}\langle u\rangle$. It is obvious moreover that any constant element, which satisfies $\dot{y} = 0$, belongs to $\mathfrak{A}$.

Take two distinct points $(\tau, u), (\tau', u') \in \mathcal{I} \times \mathcal{C}$. If $\tau \neq \tau'$, then $y = t$, which satisfies $\dot{y} = 1$, separates the two points. If $\tau = \tau'$, then assume that $u \neq u$ on the interval $[0, \tau]$. It follows from Lerch's theorem (Lerch, 1903) (see also Mikusiński, 1983) that there exists a non-negative integer ν such that

$$\int_0^t \sigma^\nu u(\sigma) d\sigma \neq \int_0^t \sigma^\nu u'(\sigma) d\sigma.$$

The classic Cauchy formula demonstrates the existence of a non-negative integer ν such that y, which satisfies $y^{(\nu)} = u$, separates $(\tau, u), (\tau, u')$.

This proof, which mimics to some extent Fliess (1976, 1981) (see also Sussmann, 1976), shows that $\mathfrak{A}$ is dense in $\mathfrak{S}$.

Appendix B. Justification of the ultra-local model

Assume that our SISO system is 'well' approximated by a system described by Equation (B2). Let ν be a non-negative integer such that

$$\frac{\partial E}{\partial y^{(\nu)}} \not\equiv 0.$$

The implicit function theorem yields then locally

$$y^{(\nu)} = \mathcal{E}(y, \dot{y}, \ldots, y^{(\nu-1)}, y^{(\nu+1)}, \ldots, y^{(a)}, u, \dot{u}, \ldots, u^{(b)}).$$

It may be rewritten as Equation (1).