HAL
open science

Stabilization of Tollmien-Schlichting waves by finite amplitude optimal streaks in the Blasius boundary layer

Carlo Cossu, L. Brandt

To cite this version:

Carlo Cossu, L. Brandt. Stabilization of Tollmien-Schlichting waves by finite amplitude optimal streaks in the Blasius boundary layer. Physics of Fluids, 2002, 14 (8), pp.L57-L60. 10.1063/1.1493791 . hal-01024915

HAL Id: hal-01024915

https://polytechnique.hal.science/hal-01024915v1

Submitted on 5 Sep 2014

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

AIP | Physics of Fluids

Stabilization of Tollmien–Schlichting waves by finite amplitude optimal streaks in the Blasius boundary layer

Carlo Cossu and Luca Brandt

Citation: Physics of Fluids (1994-present) **14**, L57 (2002); doi: 10.1063/1.1493791
View online: http://dx.doi.org/10.1063/1.1493791
View Table of Contents: http://scitation.aip.org/content/aip/journal/pof2/14/8?ver=pdfcov
Published by the AIP Publishing

Articles you may be interested in

Detuned resonances of Tollmien-Schlichting waves in an airfoil boundary layer: Experiment, theory, and direct numerical simulation
Phys. Fluids **24**, 094103 (2012); 10.1063/1.4751246

Floquet analysis of secondary instability of boundary layers distorted by Klebanoff streaks and Tollmien–Schlichting waves
Phys. Fluids **20**, 124102 (2008); 10.1063/1.3040302

The stabilizing effect of streaks on Tollmien-Schlichting and oblique waves: A parametric study
Phys. Fluids **19**, 078103 (2007); 10.1063/1.2746047

Experimental study of the stabilization of Tollmien–Schlichting waves by finite amplitude streaks
Phys. Fluids **17**, 054110 (2005); 10.1063/1.1897377

Optimal control of Tollmien–Schlichting waves in a developing boundary layer
Phys. Fluids **13**, 2087 (2001); 10.1063/1.1378035

AIP | Journal of Applied Physics

Journal of Applied Physics is pleased to announce **André Anders** as its new Editor-in-Chief

This article is copyrighted as indicated in the article. Reuse of AIP content is subject to the terms at: http://scitation.aip.org/termsconditions. Downloaded to IP: 129.104.29.1 On: Thu, 17 Jul 2014 13:36:32

LETTERS

The purpose of this Letters section is to provide rapid dissemination of important new results in the fields regularly covered by Physics of Fluids. Results of extended research should not be presented as a series of letters in place of comprehensive articles. Letters cannot exceed four printed pages in length, including space allowed for title, figures, tables, references and an abstract limited to about 100 words. ***There is a three-month time limit, from date of receipt to acceptance, for processing Letter manuscripts. Authors must also submit a brief statement justifying rapid publication in the Letters section.***

Stabilization of Tollmien–Schlichting waves by finite amplitude optimal streaks in the Blasius boundary layer

Carlo Cossu
LadHyX, CNRS–École Polytechnique, F-91128 Palaiseau Cedex, France

Luca Brandt
Department of Mechanics, Royal Institute of Technology, S-10044 Stockholm, Sweden

(Received 4 February 2002; accepted 23 May 2002; published 24 June 2002)

In this Letter we show by numerical simulation that streamwise streaks of sufficiently large amplitude are able to stabilize Tollmien–Schlichting waves in zero pressure gradient boundary layers at least up to Re=1000. This stabilization is due to the spanwise averaged part of the nonlinear basic flow distortion induced by the streaks and occurs for streak amplitudes lower than the critical threshold beyond which secondary inflectional instability is observed. A new control strategy is implemented using optimal perturbations in order to generate the streaks. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1493791]

In boundary layer flows with low levels of background disturbances, transition is initiated by the exponential amplification of the unstable Tollmien–Schlichting (TS) waves followed by secondary instabilities and breakdown to turbulence as soon as the u_{rms} amplitude of the primary instability is of the order of 1% of the free stream velocity.[1] In the presence of free stream turbulence (FST), narrow elongated regions of alternated low and high streamwise velocity called ''streamwise streaks'' slowly oscillate in the boundary layer.[2] For sufficiently large values of FST ''bypass transition'' is observed. For intermediate levels of FST it was however surprisingly found[3] that the spatial amplification rate of small amplitude TS waves was *lower* than in the quiet case. This effect was mainly attributed[3] to the 2D mean distortion of the velocity profile observed in the presence of the FST,[4] which is produced by the nonlinear saturation of the streaks.[5] In this Letter we investigate by direct numerical simulation whether artificially induced streaks are able to completely stabilize TS waves. This could be an effective alternative way to control primary instabilities in boundary layers with low levels of ambient noise.

Numerical method. The incompressible 3D Navier–Stokes equations are integrated using a pseudospectral code described in Lundbladh *et al.*[6] The code uses Fourier expansions in the streamwise and spanwise directions and Chebyshev polynomials in the wall-normal direction. The time stepping scheme is a low storage third-order Runge–Kutta method for the nonlinear terms and a second-order Crank–Nicolson method for the linear terms. Dealiasing is used in the streamwise and spanwise directions. Two types of simulations are performed in the present study: in the case of ''spatial simulations,'' i.e., simulations of non-parallel basic flows, a fringe region is employed to enforce inflow and outflow boundary conditions in a periodic domain; in the case of ''temporal'' simulations a volume force is used to keep the basic flow parallel.

Basic flow. The basic flows we consider are zero pressure gradient boundary layers with steady, nonlinearly saturated, spanwise periodic streaks of different amplitudes. We use optimal perturbations[7] in order to generate the streaks with minimum input energy. These optimal perturbations consist of vortices aligned in the streamwise direction which, owing to the ''lift-up'' effect,[8] have the highest potentiality for spatial transient growth.[9,10] As in Ref. 5, the optimal perturbation computed by Andersson *et al.*[9] is used as inflow condition close to the leading edge and its downstream evolution is followed until nonlinear saturation for different initial amplitudes. For the computation of the basic flows we use a box with inlet at Re=468.5 and dimensions of $1128\,\delta_{*0}\times 20\,\delta_{*0}\times 12.83\,\delta_{*0}$, in the streamwise, wall-normal and spanwise directions, respectively, where $576\times 65\times 32$ collocation points are used. We denote by δ_{*0} the boundary layer thickness at the inlet. The spanwise extension of the domain corresponds to one wavelength of the optimally growing streaks. Denoting by x, y, and z the streamwise, wall-normal, and spanwise coordinates, respectively, we use the following definition of the streak amplitude:[5] $A_s(x)=[\max_{y,z}(U-U_B)-\min_{y,z}(U-U_B)]/2U_\infty$, where U_∞ is the free stream velocity, $U_B(x,y)$ is the Blasius solution, and $U(x,y,z)$ is the streamwise velocity of the streak. We list in Table I the five different cases considered. Case A is nothing but the Blasius boundary layer without streaks. In Fig. 1 we display the evolution of the amplitude of the streaks B,C,D,E versus the Reynolds number $\mathrm{Re}=U_\infty\delta_*/\nu$ based on local boundary

 © 2002 American Institute of Physics

This article is copyrighted as indicated in the article. Reuse of AIP content is subject to the terms at: http://scitation.aip.org/termsconditions. Downloaded to IP: 129.104.29.1 On: Thu, 17 Jul 2014 13:36:32

TABLE I. Streak amplitude for the computed basic flows.

Case	Inlet A_s	Maximum A_s	A_s at Re=1047
A	0.0000	0.0000	0.0000
B	0.0618	0.1400	0.1396
C	0.0927	0.2018	0.2017
D	0.1235	0.2558	0.2558
E	0.1695	0.3199	0.3174

layer thickness δ_*. It has recently been shown[5] that optimal streaks with amplitude $A_s > 0.26$ are subject to secondary inflectional instabilities. Streak E is therefore secondarily unstable.

Spatial stability. In the same spirit as Boiko *et al.*,[3] we first test the spatial stability of the computed basic flows to two-dimensional harmonic perturbations of dimensionless frequency $F = 2\pi f \nu / U_\infty^2$. The same computational parameters adopted in the evaluation of the basic flows are used in this type of simulation. The perturbation is induced by a two-dimensional time periodic volume force localized at the inlet position, extending up to Re$\sim$480, of amplitude small enough to ensure a linear evolution of the perturbations. The computations were carried on for sufficiently large times to achieve converged time periodic solutions in all the computational domain. In Fig. 2 we show the downstream development of the amplitude, based on the norm $[\int_0^\infty (u'^2 + v'^2 + w'^2) dy]^{1/2}/2U_\infty$, of two-dimensional waves at the frequency $F = 1.316\,10^{-4}$ of the forcing. In the Blasius boundary layer (case A) the perturbations decay until they reach branch I of the linear neutral stability curve situated at Re $=635$ in the parallel flow approximation. After, they begin to grow until branch II is reached at Re$=1000$. When the basic flow contains a low amplitude streak (case B) an unstable domain still exists but the growth of the TS waves is attenuated in a way similar to that observed in the presence of free stream turbulence.[3] Case C presents a region of marginal stability around Re$=750$, where the streak amplitude is about 0.17. In the case of largest amplitude streaks (case D), the forced TS waves are stable. Similar results apply to forcing frequencies $F = 1.6\,10^{-4}$ and $F = 2.0\,10^{-4}$.

Temporal stability. In the spatial simulations we considered only some frequencies F and we forced only 2D perturbations. To verify the behavior of more general 3D perturbations of any frequency, we decided to investigate the temporal stability of the parallel basic flows $(U(y,z),0,0)$

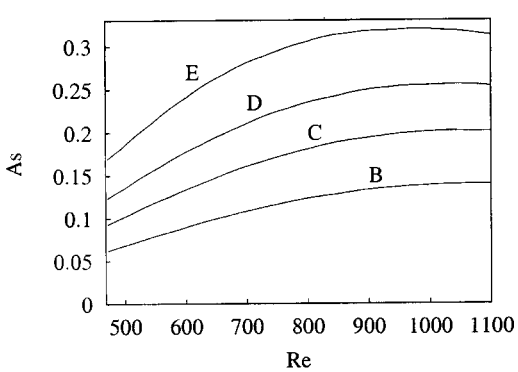

FIG. 1. Streamwise spatial evolution of the amplitude of streaks B, C, D and E.

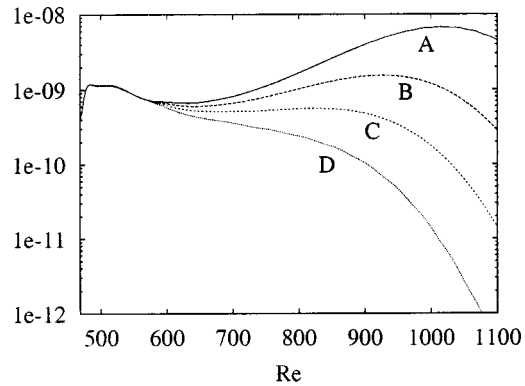

FIG. 2. Spatial evolution of the amplitude of 2D perturbations in the Blasius boundary layer without streaks (case A) and with streaks of increasing amplitude (cases B–D).

obtained by extracting the streak velocity profiles at the streamwise station corresponding to Re$=1047$. As shown in Fig. 1, at that streamwise station the streak amplitude is very slowly varying in the streamwise direction. For this type of simulations, the computational domain measures $1200\,\delta_* \times 9\,\delta_* \times 5.74\delta_*$ in the streamwise, wall-normal and spanwise direction, respectively, where $1024\times97\times32$ collocation points are used. In order to consider all the wavenumbers and frequencies at once, we study the flow response to an impulse-like initial condition. This initial condition, already used in other studies,[11–13] is given by $(u_0, v_0, w_0) = (U, \partial\psi_0/\partial z, -\partial\psi_0/\partial y)$ with $\psi_0 = P_0\, \overline{x}\overline{y}^3\overline{z}\, e^{-(\overline{x}^2+\overline{y}^2+\overline{z}^2)}$, where $\overline{x} = (x - x_0)/p_x$, $\overline{y} = y/p_y$, and $\overline{z} = (z - z_0)/p_z$. We use an initial disturbance amplitude P_0 sufficiently small to ensure a linear development of the perturbations and length scales $p_x = 5\,\delta_*$, $p_y = 2\,\delta_*$, and $p_z = 1.5\,\delta_*$ small enough to reproduce a localized impulse within the limits of a good resolution in the truncated spectral space of our numerical simulations. In order to avoid any particular spanwise symmetry of the solution, the initial disturbance is centered off-axis, around $z_0 = -2\,\delta_*$. The temporal evolution of the rms perturbation kinetic energy E, integrated over the whole computational domain, is shown in Fig. 3. Since all the modes are excited by the initial condition, any unstable mode would emerge after an initial transient. The Blasius boundary layer (case A) is linearly unstable at Re$=1047$ and TS waves

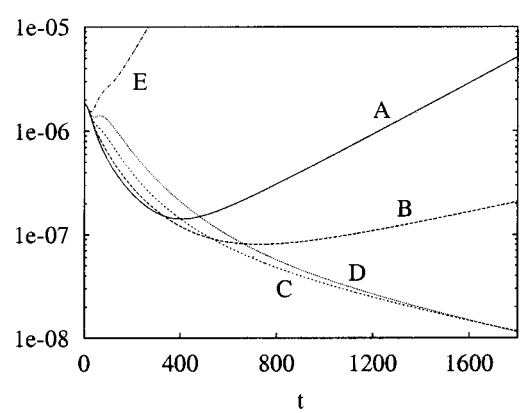

FIG. 3. Temporal evolution of the rms perturbation energy at Re$=1047$ for the Blasius boundary layer without streaks (case A), and with streaks of increasing amplitude (cases B, C, D, and E).

This article is copyrighted as indicated in the article. Reuse of AIP content is subject to the terms at: http://scitation.aip.org/termsconditions. Downloaded to IP: 129.104.29.1 On: Thu, 17 Jul 2014 13:36:32

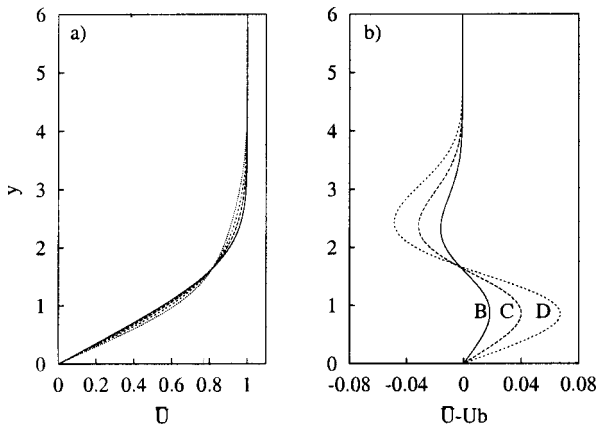

FIG. 4. (a) Spanwise averaged streamwise velocity profiles $\overline{U}(y)$, at Re $=1047$, of the Blasius boundary layer (solid line) and of the streaky boundary layers B, C, and D. (b) Corresponding spanwise averaged basic flow distortions $\overline{\Delta U}(y)=\overline{U}(y)-U_B(y)$.

emerge clearly at times $t>600$; the perturbation growth is exponential, with values in agreement with the results of the linear Orr–Sommerfeld analysis. The low amplitude streak (case B) is also unstable but with a lower amplification rate. Streaks with larger amplitude (cases C and D) are stable and, after the initial transient, perturbations decay for large times. A too large amplitude of the streaks (case E), however, leads to inflectional instability.[5]

Interpretation. We now try to isolate the mechanism responsible for the observed stabilization of the Blasius boundary layer. The basic flow distortion $\Delta U(y,z)=U(y,z)-U_B(y)$ can be separated into its spanwise averaged part $\overline{\Delta U}(y)$ and its spanwise varying part $\widetilde{\Delta U}(y,z)=\Delta U(y,z)-\overline{\Delta U}(y)$. Note that nonlinear effects are essential to generate $\overline{\Delta U}(y)$. In Figs. 4(a) and 4(b) we reproduce the spanwise averaged velocity $\overline{U}(y)=U_B(y)+\overline{\Delta U}(y)$ of the basic solutions at Re$=1047$ and the corresponding $\overline{\Delta U}(y)$. It can be seen how the increase of the streak amplitude leads to fuller $\overline{U}$-profiles and should thus have a stabilizing effect on the TS waves.[3] On the other hand, the term $\widetilde{\Delta U}(y,z)$ leads to velocity profiles which, based on a 2D local stability analysis,[14] are more unstable, at some spanwise stations z, than the Blasius profile. The results of the complete 3D stability analysis, displayed in Fig. 3, may be interpreted as follows: without streaks (case A), the Blasius profile is unstable to TS waves due to a viscous instability. Low amplitude streaks (case B) are neither able to stabilize the viscous instability with $\overline{\Delta U}(y)$, nor to create, with $\widetilde{\Delta U}(y,z)$, shears strong enough to support fully 3D inflectional instabilities. For moderate streak amplitudes (cases C and D), $\overline{\Delta U}(y)$ is able to completely stabilize the flow but, for too large amplitudes (case E) the term $\widetilde{\Delta U}(y,z)$ supports inflectional instabilities and the basic flow becomes unstable again. To confirm this interpretation we performed 3D temporal simulations for the artificial basic flows $\tilde{\mathrm{D}}$ and $\overline{\mathrm{D}}$ obtained by considering only the spanwise varying or the spanwise uniform part of the basic flow distortion, i.e., $U_{\tilde{\mathrm{D}}}=U_B+\widetilde{\Delta U}$ and $U_{\overline{\mathrm{D}}}=U_B+\overline{\Delta U}$. The temporal evolutions of the rms perturbation energies pertaining to these basic flows and to the complete streak D are

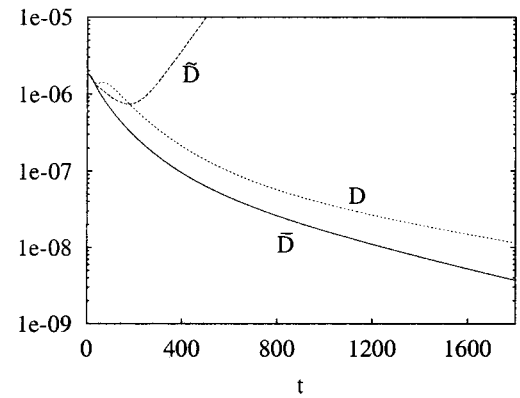

FIG. 5. Temporal evolution of the rms perturbation energy at Re$=1047$ for the complete streak D and for the artificial basic flows $\tilde{\mathrm{D}}$ and $\overline{\mathrm{D}}$.

displayed in Fig. 5; the figure reveals both the destabilizing role of $\widetilde{\Delta U}(y,z)$ and, more importantly, the dominating stabilizing action of $\overline{\Delta U}(y)$, to which the global stabilization of TS waves can finally be attributed. The same result applies to case C.

Boundary layer control. We have shown that it is possible to modify an unstable Blasius boundary layer into a stable streaky flow introducing streamwise vortices near the leading edge. The "actuator" input energy is minimized by the choice of an optimal forcing and is of $O(1/\mathrm{Re})$ when compared to the streak energy.[9,10] Moreover, it was recently observed[5] that the steady saturated streaks induced by the optimal vortices maintain an almost constant amplitude for a large distance downstream. The stabilization is obtained, in open loop, through a modification of the basic flow induced by actuators situated upstream of the unstable domain. The local skin friction coefficient of the streaky flow is increased, at worst, by less than 20%, compared to the Blasius boundary layer. We tested this control strategy for Reynolds numbers up to 1047 for a fixed spanwise periodicity of the streaks. For larger Reynolds number probably larger streak amplitudes would be necessary to achieve stabilization. The maximum allowed amplitude is however limited by the appearance of secondary instabilities on the streaks. We are therefore currently identifying stabilizing streak amplitude thresholds for a wider range of Reynolds numbers and of spanwise wavelength.

ACKNOWLEDGMENTS

The authors kindly thank P. Huerre for his comments on this Letter. The computational facilities were provided by CNRS/IDRIS. L.B. acknowledges financial support by École Polytechnique (EGIDE) and VR (Vetenskapsrådet) during his stay at LadHyX, where this work was performed.

[1] T. Herbert, "Secondary instability of boundary-layers," Annu. Rev. Fluid Mech. **20**, 487 (1988).
[2] P. S. Klebanoff, "Effect of free-stream turbulence on the laminar boundary layer," Bull. Am. Phys. Soc. **10**, 1323 (1971).
[3] A. V. Boiko, K. J. A. Westin, B. G. B. Klingmann, V. V. Kozlov, and P. H. Alfredsson, "Experiments in a boundary layer subjected to free stream turbulence. Part 2. The role of TS waves in the transition process," J. Fluid Mech. **281**, 219 (1994).

This article is copyrighted as indicated in the article. Reuse of AIP content is subject to the terms at: http://scitation.aip.org/termsconditions. Downloaded to IP: 129.104.29.1 On: Thu, 17 Jul 2014 13:36:32

[4]K. J. A. Westin, A. V. Boiko, B. G. B. Klingmann, V. V. Kozlov, and P. H. Alfredsson, ''Experiments in a boundary layer subjected to free stream turbulence. Part 1. Boundary layer structure and receptivity,'' J. Fluid Mech. **281**, 193 (1994).

[5]P. Andersson, L. Brandt, A. Bottaro, and D. Henningson, ''On the breakdown of boundary layers streaks,'' J. Fluid Mech. **428**, 29 (2001).

[6]A. Lundbladh, S. Berlin, M. Skote, C. Hildings, J. Choi, J. Kim, and D. S. Henningson, ''An efficient spectral method for simulation of incompressible flow over a flat plate,'' Technical Report KTH/MEK/TR–99/11–SE, KTH, Department of Mechanics, Stockholm (1999).

[7]P. J. Schmid and D. S. Henningson, *Stability and Transition in Shear Flows* (Springer, New York, 2001).

[8]M. T. Landahl, ''A note on an algebraic instability of inviscid parallel shear flows,'' J. Fluid Mech. **98**, 243 (1980).

[9]P. Andersson, M. Berggren, and D. Henningson, ''Optimal disturbances and bypass transition in boundary layers,'' Phys. Fluids **11**, 134 (1999).

[10]P. Luchini, ''Reynolds-number independent instability of the boundary layer over a flat surface. Part 2: Optimal perturbations,'' J. Fluid Mech. **404**, 289 (2000).

[11]K. S. Breuer and J. H. Haritonidis, ''The evolution of a localized disturbance in a laminar boundary layer. Part 1. Weak disturbances,'' J. Fluid Mech. **220**, 569 (1990).

[12]D. S. Henningson, A. Lundbladh, and A. V. Johansson, ''A mechanism for bypass transition from localized disturbances in wall-bounded shear flows,'' J. Fluid Mech. **250**, 169 (1993).

[13]K. H. Bech, D. S. Henningson, and R. A. W. M. Henkes, ''Linear and nonlinear development of localized disturbances in zero and adverse pressure gradient boundary layers,'' Phys. Fluids **10**, 1405 (1998).

[14]D. W. Wundrow and M. E. Goldstein, ''Effect on a laminar boundary layer of small-amplitude streamwise vorticity in the upstrem flow,'' J. Fluid Mech. **426**, 229 (2001).

This article is copyrighted as indicated in the article. Reuse of AIP content is subject to the terms at: http://scitation.aip.org/termsconditions. Downloaded to IP: 129.104.29.1 On: Thu, 17 Jul 2014 13:36:32