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Sébastien Corde, Kim Ta Phuoc

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S. Corde and K. Ta Phuoc

Laboratoire d'Optique Appliquée, ENSTA ParisTech—CNRS UMR7639—École Polytechnique, Chemin de la Hunière, 91761 Palaiseau, France

Laser-plasma accelerators have become compact sources of ultrashort electron bunches at energies up to the gigaelectronvolt range thanks to the remarkable progress made over the past decade. A direct application of these electron bunches is the production of short pulse x-ray radiation sources. In this letter, we study a fully optically driven x-ray source based on the combination of a laser-plasma accelerator and a plasma wave undulator. The longitudinal electric field of a laser-generated plasma wave is used to wiggle electrons transversally. The period of this plasma undulator being equal to the plasma wavelength, tunable photon energies in the 10 keV range can be achieved with electron energies in the 100–200 MeV range. Considering a 10s TW class femtosecond laser system, undulators with a strength parameter $K \sim 0.5$ and with about ten periods can be combined with a laser-plasma accelerator, resulting in several 10^{-2} emitted x-ray photons per electron. © *2011 American Institute of Physics*. [doi:10.1063/1.3569827]

I. INTRODUCTION

Laser wakefield acceleration (LWFA) has been one of the most developed topics of high intensity laser-plasma interaction during the past ten years. It has led to a number of theoretical and experimental efforts resulting in remarkable results. In LWFA, electrons are accelerated from rest to relativistic energies thanks to the strong longitudinal electric field (100s GeV/m) of a large amplitude plasma wave driven in the wake of an intense femtosecond laser pulse. Electron beams currently produced are quasimonoenergetic,[1–3] with an energy spread down to below the percent,[4] an energy up to the gigaelectronvolts,[5] a charge in the 10–100 pC range, a divergence of a few millirads, a duration in the few femtoseconds range,[6] and a normalized emittance in the $\pi \cdot \text{mm} \cdot \text{mrad}$ range.

In the interaction regimes of laser wakefield electron acceleration, there usually occurs the emission of intense and collimated femtosecond x-ray bursts. Several mechanisms, based on the radiation from relativistic electrons from plasmas, have been studied as sources of femtosecond x-rays: the nonlinear Thomson scattering of a laser beam by plasma electrons,[7,8] the Betatron oscillations of accelerated electrons in the bubble regime,[9] the Thomson backscattering of a laser beam by laser-accelerated electrons,[10] and the oscillations of laser-accelerated electrons in a periodical structure of magnets (the conventional meter-scale undulators used in synchrotron facilities).[11,12] These high-brightness, compact, and ultrashort x-ray sources can potentially have many applications in fundamental research as well as in the industrial or medical domains. However, even if these sources can already cover a spectral range from a few electron volts (conventional undulators) to a few 100 keV (Thomson backscattering), there exists a gap in the 10 keV range where none of these sources are efficient. Only the K_α source emits in this energy range but it is a fully isotropic radiation and has a duration of a few 100s fs. The production of a radiation beam with a few femtoseconds duration at this energy is therefore

particularly important for applications. Indeed, most of the applications require a source with an Ångström wavelength and a femtosecond duration, respectively, the typical interatomic distances and time scales of fundamental processes in matter.

In this letter, we discuss the possibility to produce a fully optically driven femtosecond and tunable x-ray source in the 10 keV range by using a laser-generated plasma wave as an undulator for a laser-accelerated electron bunch. The concept of the plasma wave undulator was originally proposed in the context of the realization of compact short wavelength free electron lasers from conventional accelerators.[13,14] Indeed, such an undulator has a very small period compared to conventional undulators[15] and allows to produce short wavelength free electron radiation with moderate electron energies. Further studies on the plasma wave undulator were performed a few years later,[16–19] discussing in more detail the classical spontaneous emission. Electron trajectories as well as the incoherent radiation they emit were analytically or numerically calculated using ideal linear plasma waves. In the following, we will study the utility of the plasma wave undulator for laser-plasma accelerators. The interest of this study is motivated by the active research on femtosecond x-ray sources and by the fact that laser-plasma accelerators have become efficient and reliable at the 100 MeV energy level. We will show that, at these electron energies, high brightness and tunable undulator type radiation can be produced in the relevant 10 keV range. Our description does not rely on ideal linear plasma wave. Instead, we first propose a concise analytical analysis of the nonlinear one-dimensional plasma wave undulator for estimation of the radiation properties, and then fully describe the realistic plasma wave generation using quasistatic particle code simulations.[20] These realistic plasma waves are then used to calculate numerically electron trajectories and the emitted x-ray radiation. The consideration of realistic plasma waves is particularly important since for the considered amplitudes they become nonlinear,
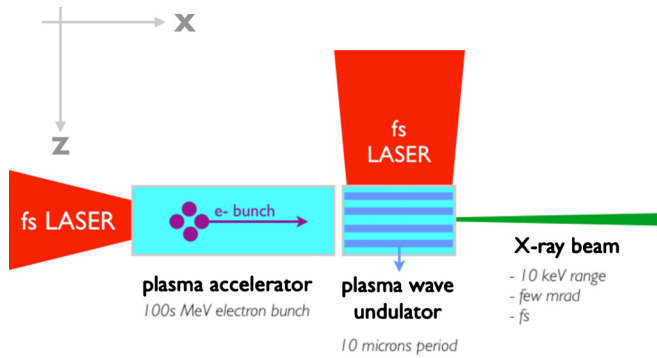
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FIG. 1. (Color online) Principle of the plasma undulator source combined with a laser-plasma accelerator. An electron bunch is created and accelerated at the interaction of an intense femtosecond laser with a millimeter scale plasma. Once accelerated, electrons propagate and are wiggled in a plasma wave. Their relativistic oscillation motion results in the emission of an x-ray beam.

with possible damping and breaking of the plasma wave, and strong modification of the driving laser pulse along its propagation in the plasma. All these effects are taken into account in the used quasistatic particle code Wake.[20]

A plasma wave corresponds to the coupled propagation of the electron density n_e and the scalar/electric potential V. For nonlinear multidimensional plasma waves, the charge current $\vec{j}_e$ and the magnetic field $\vec{B}$ also appear in their description. Several routes exist to produce plasma waves. They have been explored in order to produce intense electric fields for electron acceleration in plasmas. Large amplitude plasma waves can be generated through the beating of two copropagating long laser pulses satisfying the condition $\omega_2 - \omega_1 = \omega_p$ [$\omega_p = (n_e e^2/m\epsilon_0)^{1/2}$ is the plasma frequency, ω_1 and ω_2 the laser frequencies of each laser pulse], the self-modulation instability [a single long laser pulse breaks in a pulse train of duration $\tau \sim \lambda_p/c$, where $\lambda_p = 2\pi c/\omega_p$ is the plasma wavelength], and the resonant case where the laser pulse has a duration $\tau \sim \lambda_p/c$. These methods have been demonstrated to be efficient for the generation of electric fields $\vec{E} = -\vec{\nabla}V$ in the 100s GeV/m range (it has to be compared with the highest achievable electric field by conventional approaches, which is in the 10s MeV/m range). In the following, we will consider the generation of plasma waves by a resonant excitation.

The schematic of the x-ray source studied is presented in Fig. 1. Electrons are first accelerated to relativistic energies in a laser-plasma accelerator (along the $\hat{x}$ direction) and then propagated in a plasma wave traveling in the $\hat{z}$ direction. As they propagate in the plasma wave, they experience successively an electric field directed toward the $\hat{z}$ positive values and toward the $\hat{z}$ negative values. As a consequence, they oscillate in the $\hat{z}$ direction with a significant transverse acceleration. Since both the plasma wave and the electron bunch travel at almost the speed of light, the period of the electron motion is equal to the plasma wave period, i.e., approximately the plasma wavelength (which is 10.6 μm for an electron density of 10^{19} cm^{-3}). The result of this relativistic oscillatory motion is the emission of an x-ray beam by the electron bunch. Experimentally, this can be realized by using

two synchronized laser pulses. The first laser pulse, directed along the $\hat{x}$ direction, is used to create the plasma accelerator, producing a relativistic electron bunch. The second laser pulse, propagating in the transverse direction $\hat{z}$, is used to create the plasma wave undulator.

II. ANALYTICAL ANALYSIS

The radiation produced in the plasma undulator is a relativistic moving charge radiation and its features depend on the electron motion. In order to derive estimations for the electron orbits driven by a plasma wave undulator, we consider a one-dimensional plasma wave described by the normalized electric potential $\phi(\xi) = eV/mc^2$ only depending on the comoving variable $\xi = z - v_p t$ (quasistatic approximation), v_p being the plasma wave phase velocity and supposedly equal to the laser group velocity.

We assume an electron from a laser-plasma accelerator with an initial energy $\mathcal{E} = \gamma_i mc^2$ (γ_i is the initial relativistic factor) and a velocity directed along the $\hat{x}$ axis. The electron enters transversally in the one-dimensional (1D) plasma wave potential $\phi(\xi)$. The equation of the test electron motion is

$$\frac{d\vec{p}}{dt} = \vec{\nabla}\phi, \tag{1}$$

where all quantities are normalized by the choice $m = c = e = \omega_p = 1$. The Hamiltonian describing the dynamic of the test electron in the plasma wave is

$$\mathcal{H}(\vec{r}, \vec{P}, t) = \gamma - \phi = \sqrt{1 + (\vec{P} + \vec{a})^2} - \phi, \tag{2}$$

where $\vec{P} = \vec{p} - \vec{a}$ is the canonical momentum. In our case, the canonical and kinematic momentum $\vec{p}$ are equal because the vector potential $\vec{a} = 0$. $\mathcal{H}$ does not depend on the x and y coordinates. This implies the conservation of p_x and p_y. Furthermore, it depends on t and z only through $\xi = z - \beta_p t \simeq z - t$. This gives the following constant of motion:

$$\gamma - \phi - p_z = \text{const.} \tag{3}$$

Using the initial conditions for the test electron: $x = y = z = p_y = p_z = 0$, $p_x = \sqrt{\gamma_i^2 - 1}$, $\phi = 0$, the above constants of motion and the relation $\gamma^2 = 1 + p^2$, we obtain

$$p_z = -\phi + \frac{\phi^2}{2\gamma_i} + o\left(\frac{\phi^2}{\gamma_i}\right), \tag{4}$$

where we develop expressions using the approximation $\phi \ll \gamma_i$, which is always satisfied (the normalized potential cannot exceed a few units in realistic plasma waves, whereas γ_i is superior to 100 for our case).

The electron motion consists of a drift in the $\hat{x}$ direction combined with an oscillation in the transverse direction $\hat{z}$. The plasma wave $\phi(\xi)$ acts as an undulator with a period $\lambda_u \simeq \lambda_p$. This period depends on the plasma density and can be controlled. In practical units, it is given by $\lambda_u[\mu m] = 3.34 \times 10^{10}/\sqrt{n_e[\text{cm}^{-3}]}$. A second relevant parameter characterizing the plasma undulator is its strength parameter K. It is defined as the product of the electron energy γ by the maximum angle of the trajectory ψ (with respect to the $\hat{x}$

axis): $K = \gamma\psi$. Because $\psi = dz/dx_{max} \simeq dz/dt_{max} = p_{z\,max}/\gamma$, from Eq. (4), the strength parameter K of the plasma wave undulator reads

$$K = \gamma\psi = p_{z\,max} = \phi_0,$$ (5)

where ϕ_0 is the amplitude of the plasma wave potential $\phi(\xi)$. In the linear limit $a_0^2 \ll 1$, the plasma wave reads $\phi(\xi) = \sqrt{\pi}a_0^2(\omega_p\tau/4)\exp(-\omega_p^2\tau^2/4)\sin(k_p\xi)$, for a Gaussian laser pulse with duration τ at $1/e^2$ in intensity, and if we approach the nonlinear regime, we can reach $K = \phi_0 \lesssim 1$.

The radiation emitted by an electron traveling in the plasma wave undulator is a moving charge emission described by the general expression giving the radiation emitted by an electron, in the direction of observation $\vec{n}$, as a function of its position, velocity, and acceleration along the trajectory,[21]

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2}{16\pi^3\epsilon_0 c} \left| \int_{-\infty}^{+\infty} e^{i\omega(t-\vec{n}\cdot\vec{r}(t)/c)} \frac{\vec{n} \times [(\vec{n}-\vec{\beta}) \times \dot{\vec{\beta}}]}{(1-\vec{\beta}\cdot\vec{n})^2} dt \right|^2.$$ (6)

This expression represents the energy radiated within a spectral band $d\omega$ centered on the frequency ω and a solid angle $d\Omega$ centered on the direction of observation $\vec{n}$. Here, $\vec{r}(t)$ is the electron position at time t, $\vec{\beta}$ is the velocity of the electron normalized to the speed of light c, and $\dot{\vec{\beta}} = d\vec{\beta}/dt$ is the usual acceleration divided by c. The analysis of this general expression results in analytical expressions describing the radiation features as a function of the parameters γ_i, λ_u, and K. In the laboratory frame, the radiation emitted by a relativistic electron at a given time is directed in the direction of its velocity $\vec{v}$, within a cone of typical angle $1/\gamma$. The angular distribution of the radiation depends on the electron orbit. For a trajectory confined in the plane $(\hat{x}, \hat{z})$, which is the case for a 1D plasma wave, the radiation is emitted in a cone of half angle $\theta = 1/\gamma$ for $K < 1$, while for $K > 1$ the angular distribution is increased in the direction of the oscillation $\hat{z}$ with a half angle of K/γ. The electron motion being periodic with a spatial period λ_u, the spectrum of the radiation consists of the fundamental wavelength $\lambda = \lambda_u/2\gamma^2(1+K^2/2+\gamma^2\theta^2)$ and its harmonics, where θ is the observation angle with respect to $\hat{x}$. For the undulator limit $K \ll 1$, i.e., small transverse amplitudes of oscillation, the spectrum contains only the fundamental wavelength, and harmonics start to appear when $K \to 1$. In the wiggler limit $K \gg 1$, the spectrum contains many closely spaced harmonics extending up to a critical wavelength $\lambda_c = 2\lambda_u/3K\gamma^2$. For the case of the plasma wave, $K \lesssim 1$ and the radiation is produced in the undulator regime at the fundamental wavelength and possibly a few harmonics. The spectral thickness of the fundamental wavelength and its harmonics depend on the number of oscillation periods N. It is given by $\Delta\lambda_n/\lambda_n = 1/(nN)$ where the index n indicates the harmonic order. Note that the spectrum is broadened with a decreasing energy when observed off axis. The total number of emitted photons depends on the electron energy γ, the K parameter, and the number of oscillations N. Over one oscillation period and for $K \ll 1$, it is given by $N_X = 2\pi/3\alpha K^2$ (at the mean energy $\langle\hbar\omega\rangle = \gamma^2 hc/\lambda_u$),[21] where $\alpha = e^2/4\pi\epsilon_0\hbar c$ is the fine structure constant. Finally, the duration of the emitted radiation pulse is sensibly equal to the duration of the electron bunch. In the case of laser-plasma acceleration, the duration of the electron bunch can be as small as a few femtoseconds.[6]

For example, if we consider a 150 MeV electron traveling in a plasma wave undulator with $K = 0.5$ and a period $\lambda_u = 10$ μm, the produced radiation has an on-axis fundamental energy $E_X \simeq 21$ keV. The half angle of the x-ray beam divergence is $\theta \simeq 3$ mrads and the expected number of photons produced per electron and per period is $\simeq 4 \times 10^{-3}$. Assuming 10^8 electrons and 10 periods, the total number of photons is $\simeq 4 \times 10^6$. This photon energy range is not covered by the existing laser produced x-ray sources. In addition, with the present methods of wakefield acceleration, the electron energy can typically be tuned from 50 to 200 MeV,[22] which corresponds to x-ray energy from ~2 to 37 keV.

III. NUMERICAL ANALYSIS

The above description assumes an idealized situation where the plasma wave is 1D and the electron motion is strictly periodic. A more precise description based on numerical simulations is presented in the following.

The generation of the plasma wave by a resonant excitation has been calculated using the Wake code.[20] Wake is a two-dimensional moving window particle code describing laser-plasma interaction in the short pulse relativistic regime. It allows to calculate the propagation of a relativistic laser pulse in an underdense plasma and its wake fields. To speed up the calculation time compared to PIC (particle in cell) simulations, Wake assumes that the laser pulse does not evolve significantly during the electron transit time through the laser pulse (quasistatic approximation), and it uses averaged equation of motions for the particles (average over the laser period) so that it is only necessary to resolve the plasma time scale, and not the laser time scale. As a consequence, the code cannot describe high energy electrons moving in the forward direction with $\gamma > \omega_0/\omega_p$, where ω_0 is the laser frequency.

The wakefield maps obtained from Wake are then used to calculate the orbits of test electrons in the plasma wave undulator. From the electron trajectories, we compute the x-ray radiation properties using Eq. (6).

We have considered a laser pulse, propagating in the $\hat{z}$ direction, focused on a gas jet with a density of $n_e = 10^{19}$ cm^{-3}. The focal spot is chosen to be elliptical, with a waist larger in the $\hat{x}$ direction than in the $\hat{y}$ direction in order to increase the overlap of electrons with the plasma wave. The laser waist equals 11 μm in the $\hat{y}$ direction, the pulse duration is 30 fs at $1/e^2$ in intensity, and the amplitude of the normalized vector potential of the laser pulse is $a_0 = 1.2$. After 0.5 mm of propagation in the plasma, the Wake simulation [run in the two dimensions $(\hat{z}, \hat{y})$] shows that the amplitude of the plasma wave attains $\phi_0 \simeq 0.5$, as displayed on Fig. 2. The maps of the wake fields E_z, E_y, and B_x, after 0.5 mm of propagation in the plasma, characterize the plasma wave undulator. To take into account the finite dimension of
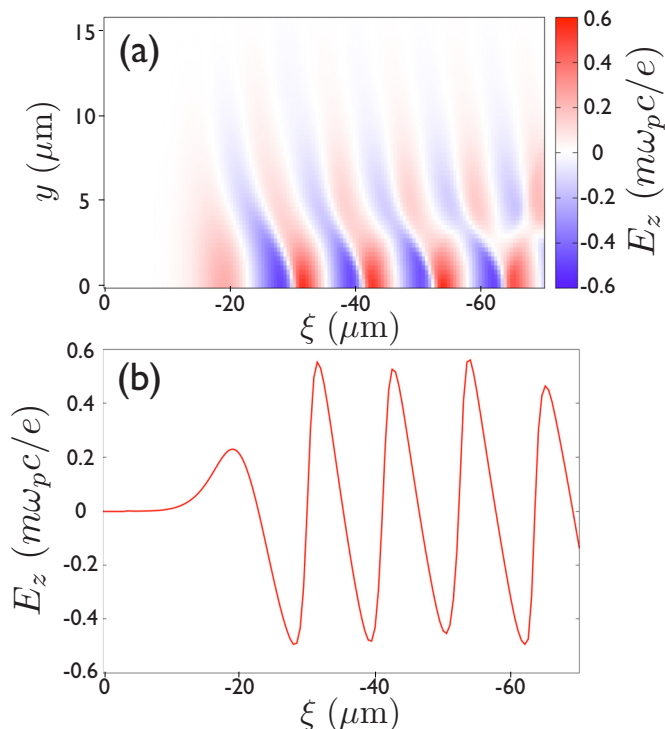
FIG. 2. (Color online) The map of the field E_z obtained from the Wake code is displayed in (a). It corresponds to the wakefield after 0.5 mm of propagation of the laser pulse in the gas jet of density $n_e = 10^{19}$ cm^{-3}. The profile $E_z(\xi)$ at $y=0$ is shown in (b).

the laser pulse and of the wake fields in the $\hat{x}$ direction, we apply a Gaussian profile with a waist of 44 μm to the wake fields for this direction.

We have injected an electron beam of energy $\mathcal{E}$ $=150$ MeV and of transverse size 7 μm, propagating in the $\hat{x}$ direction, in the plasma wave undulator characterized by the fields obtained from Wake, and we have calculated the produced radiation using Eq. (6). The electrons emit a narrow x-ray beam directed in the $\hat{x}$ direction, with a divergence of a few mrads. The angular profile of the radiation is shown in Fig. 3. Figure 4 displays the spectra of the radiation emitted on-axis as well as integrated over the angular distribution. It is in agreement with the properties of the undulator radiation, with an angle-dependent wavelength and a narrow
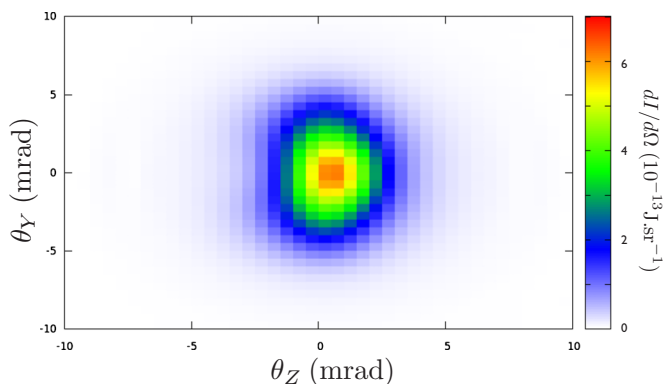

FIG. 3. (Color online) x-ray angular profile of the plasma wave undulator radiation, in J/sr and per electron.
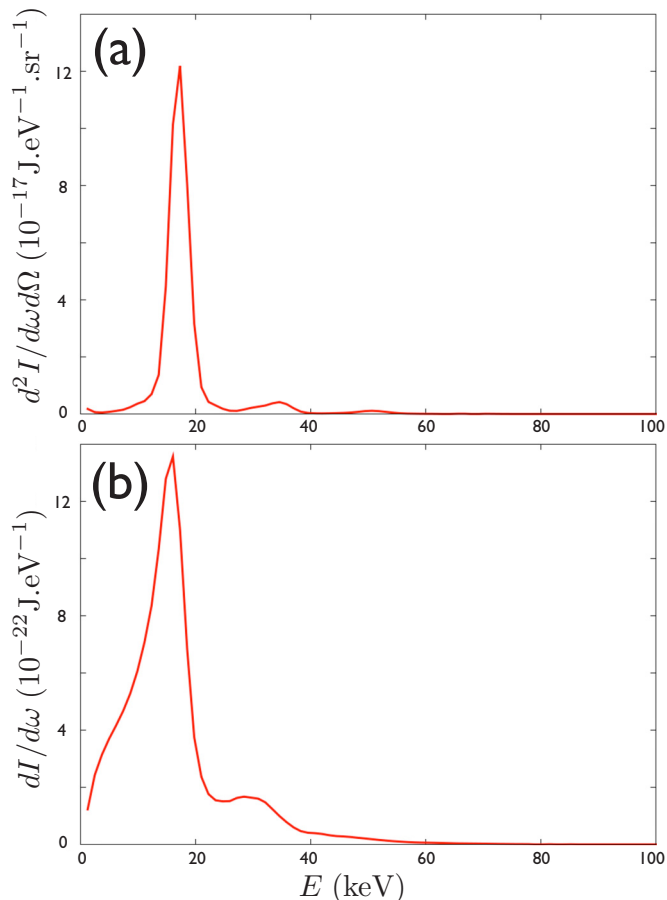

FIG. 4. (Color online) The on-axis spectrum ($\theta_Z = \theta_Y = 0$) of the plasma wave undulator radiation is displayed in (a), in J/eV/sr and per electron. The integrated spectrum over angles is shown in (b), in J/eV and per electron.

bandwidth at a given angle of observation. The central photon energy and the full width at half maximum bandwidth of the on-axis spectrum are, respectively, 17 keV and 22%. The total number of emitted photons is 10^{-2} per electron. The peak brightness associated to the on-axis spectrum of Fig. 4 is on the order of $10^{20}-10^{21}$ photons/0.1%bandwidth/s/ mm^2/mrads^2, considering 10^9 electrons, a source size of 5–10 microns, and a duration of a few femtoseconds. These results show good agreement with the estimations obtained with the analytical 1D study.

IV. CONCLUSION

In conclusion, the combination of a laser-plasma accelerator with a plasma undulator offers the possibility to produce a compact tunable source of ultrashort x-ray radiation in the 10 keV range. This source would be tunable in a wide spectral range thanks to both the tunability of the undulator period with the plasma density and the electron energy. This x-ray source exhibits the properties of undulator radiation: narrow bandwidth spectrum and high brightness.

The plasma wave undulator approach presents similarities with the Thomson scattering, and their advantages and disadvantages will be briefly discussed. In the Thomson scattering geometry, a laser pulse scatters off the electron beam from the laser-plasma accelerator. Electrons oscillate in the

laser pulse and emit radiation that can rapidly attain very high photon energies thanks to two successive Doppler shifts. The oscillation period is given by $\lambda_u \simeq \lambda_0 / (1 - \beta \cos \theta)$, where λ_0 is the laser wavelength and θ the angle between the laser and the electron beam directions (copropagating for $\theta = 0$), and the strength parameter of the oscillation is $K = a_0$ where a_0 is the normalized amplitude of the laser pulse vector potential. If we consider the highest quality electron beams from a laser-plasma accelerator produced to date, their energy is in the 100 MeV range. At these energies, the Thomson scattering is more appropriate to produce energetic photons, in the 100 keV or MeV range. Indeed, the production of x-ray radiation at 10 keV using electrons at 200 MeV requires a copropagating geometry where the laser and the electron beam make about a 10° angle. This complicates the experimental setup due to the positioning of the focusing optics. In addition, the laser scattering off the electron beam has to travel into the few millimeters gas jet, and can diffract and lose energy. Both the Thomson scattering and plasma wave undulator sources can have a strength parameter K of the same order of magnitude, $\lesssim 1$, but it is easier to reach a higher number of oscillation periods with the Thomson scattering, and so a higher number of photons and brightness.

The plasma wave undulator appears to be an interesting scheme because its period in the 10 μm range is adapted to the electron energies that are currently produced. It allows to reach the 10 keV range using electrons at a few 100s MeV. In addition, the undulator period and therefore the energy of the radiation can be easily tuned by varying the plasma density, keeping the electron beam constant. Even if the two schemes can therefore be used to access the 10 keV range and produce femtosecond x-ray radiation, the Thomson scattering looks more appropriate to access higher energies, the 100 keV range, while the plasma wave undulator is suitable to access the 10 keV range.

In this letter, we have chosen to discuss the properties of this source as it could be produced with 10–100 TW short pulse laser systems. The numerical simulations, in agreement with analytical predictions, show that a radiation beam, collimated within a few mrads, at energies in the 10 keV range, and with $\sim 10^{-2}$ photons/electron can be produced. Assuming electron bunches with 60 pC charge, the source can deliver $\sim 10^6$ photons/pulse. The source size is equal to the dimension of the electron bunch within the undulator. This depends on the distance between the electron source and the undulator. Typically, a source size on the order of 5 μm can be expected. The flux of this source is not as high as the Betatron source or the K_α source; however, this source combines the advantages of narrow band spectrum and tunability, the collimation of a few mrads, and the ultrashort pulse duration of a few femtoseconds. With more laser energy, a major improvement would be to increase the number of periods of the plasma wave undulator, by focusing on a larger waist in the direction of propagation of electrons $\hat{x}$, and by generating a plasma wave consisting of many plasma periods. Assuming a plasma wave undulator with 25 periods and with $K \simeq 0.5$, the source could reach the level of $\sim 10^{-1}$ photons/electron with an on-axis spectral bandwidth of a few percents. The use of future high repetition rate laser systems would allow to improve the averaged brightness of this source.

ACKNOWLEDGMENTS

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