Model-based versus model-free control designs for improving microalgae growth in a closed photobioreactor: Some preliminary comparisons

Sihem Tebbani¹, Mariana Titica², Cédric Join³,⁵,⁶, Michel Fliess⁴,⁵, Didier Dumur¹

Abstract—Controlling microalgae cultivation, i.e., a crucial industrial topic today, is a challenging task since the corresponding modeling is complex, highly uncertain and time-varying. A model-free control setting is therefore introduced in order to ensure a high growth of microalgae in a continuous closed photobioreactor. Computer simulations are displayed in order to compare this design to an input-output feedback linearizing control strategy, which is widely used in the academic literature on photobioreactors. They assess the superiority of the model-free standpoint both in terms of performances and implementation simplicity.

Key Words—Microalgae, photobioreactor, model-free control, intelligent proportional controller, input-output feedback linearizing controller.

I. INTRODUCTION

The production and the culture of microalgae play a growing industrial and commercial role (see, e.g., [1], [2], and the references therein). Their relationship with renewable energy and sustainable development should also be emphasized (see, e.g., [3]–[11]). The corresponding cultivation systems, which are called photobioreactors, or PBR, give rise to challenging control questions which have already attracted a lot of attention. Most of the existing academic publications are model-based (see, e.g., [12]–[20]). Among the various control techniques, which are often nonlinear, optimal control [13], predictive control [11], [12], [19]), adaptive control [16], [17]), feedback linearization [15], [18], [20]), and the use of partial differential equations [14] are perhaps the most popular ones. Although those papers are quite promising, they suffer from the great difficulty of deriving a “good” mathematical modeling of the bioprocess (see, e.g., [21]). It is due to

1) its inherent complexity,

2) its uncertain and time-varying characteristics since a life process has to be taken into account.

This communication is introducing therefore a new model-free control setting [22], which is moreover rather easy to implement both from software [22] and hardware [23] standpoints. There are many concrete applications, including some patents. We select here for obvious reasons publications that are related to biotechnology: [24]–[27].

The performances of this approach are compared to those achieved by an input-output (I/O) feedback linearization, which is among the most widely used control design in the academic literature on bioreactors [20]. The cultivation of the microalgae Chlamydomonas reinhardtii is considered here. The biomass concentration will be regulated to a target value, determined so that a high level of biomass productivity is achieved. The influence of light fluctuation on the reference tracking is also taken into account.

Our paper is organized as follows. The system and its modeling are presented in Section II. Section III displays two control strategies: an I/O feedback linearizing control and a model-free one. These strategies include two steps: (i) the choice of the setpoint that leads to a high biomass productivity, (ii) the regulation of the system around this setpoint. Numerical results are provided and discussed in Section IV. Conclusions and perspectives are developed in Section V.

II. SYSTEM DESCRIPTION AND MODELING

In the continuous operation of PBR, the reactor is continuously fed with the liquid medium culture with nutrients; the rate of outflow is equal to the rate of inflow (F) and the culture volume (V) remains constant. The manipulated variable in this case is the dilution rate (D = F/V). The microorganism population grows in the medium consuming nutrients (dissolved CO₂, nitrogen, phosphorus). In autotrophic conditions, the solely carbon source is CO₂, which is provided continuously by air enriched CO₂ bubbled in the liquid medium; its injection depends on the pH, which is maintained at the growth optimum value [15]. By this way, all liquid nutrients are provided in sufficient quantities for avoiding mineral limitations. The main factor governing the growth is the light which is the energy source for the growth. PBR surface is lighted artificially or naturally. For a given PBR geometry, light intensity distribution in the cultivation medium depends on biomass concentration as well as on optical properties of the microalgae, which are determined
by their shape and pigment content. In continuous, mineral nonlimiting cultivation conditions, the only limiting factor is the photon flux density received by the culture. Improving the light availability is thus a crucial aspect of biomass growth and process productivity. A low amount of light would decrease the growth. This is due to lack of energy necessary to fixate carbone. Excessive irradiance on the other hand would induce inhibition phenomena.

Dynamic models describing the behavior of microalgae cultures are usually a set of nonlinear ordinary differential equations. They are mainly deduced from mass balance considerations on both liquid and gaseous phases [28]. In an “optimal” system ensuring nonlimiting conditions with respect to the liquid nutrients and environmental conditions, i.e., temperature and pH, the rate of photosynthesis and productivity are determined by the light availability [29].

In this case, the model is represented by one differential equation expressing biomass concentration dynamics (1), coupled with algebraic equations giving light profile into the culture bulk (radiative model) and kinetic law yielding local photosynthetic responses by expressing growth rate as a function of local irradiance in the bulk depth. Different kinetic models are presented in the literature [21]. In this paper, a predictive model from [29] has been used as benchmark for our controllers.

The mass balance model of a continuous well-stirred PBR for biomass concentration \( X \), in kg/m\(^3\), is as follows:

\[
dX = r X - DX \tag{1}
\]

where \( rX \) is the growth rate (kg/m\(^3\)/h) and \( D \) the dilution rate (h\(^{-1}\)).

The kinetic model proposed by [29] predicts photosynthesis and respiration of microalgae from an energetic analysis.

The biomass growth rate \( rX \) is stoichiometrically linked to the net oxygen evolution rate \( \langle J_{O_2} \rangle \):

\[
rX = \frac{\langle J_{O_2} \rangle M_s X}{v_{O_2-x}} \tag{2}
\]

where \( M_s \) is the C-molar mass for the biomass (g mol\(^{-1}\)) and \( v_{O_2-x} \) the stoichiometric coefficient. \( \langle J_{O_2} \rangle \) is calculated with:

\[
\langle J_{O_2} \rangle = \frac{1}{L} \int_0^L J_{O_2}(z)dz \tag{3}
\]

where \( L \) is the total reactor depth, \( z \) is the depth of the reactor in rectangular coordinates (since the PBR here is rectangular. The PBR considered here was presented in detail in [15]), and \( J_{O_2}(z) \) represents the local specific rate of \( O_2 \) production and consumption in the reactor depth (in molO\(_2\)/kgv/h). It is the result of photosynthetic production and respiration consumption, on which evolution depends on the local irradiance:

\[
J_{O_2}(z) = \rho m \frac{K}{K + G(z)} \Phi \varepsilon_a G(z) - \frac{J_{NADH_2}}{v_{NADH_2-O_2}} \frac{K_R}{K + G(z)} \tag{4}
\]

where:

- \( G(z) \) is the local irradiance,
- \( \rho_m \) is the maximum value of the energetic yield,
- \( E_a \) is the mass absorption coefficient, linked to the pigment content,
- \( \Phi \), \( J_{NADH_2} \), and \( v_{NADH_2-O_2} \) are stoichiometric yield expressed from the stochiometric equation of biomass synthesis,
- \( K \) is the half saturation constant of photosynthesis, describing photosynthesis saturation with increasing light,
- \( K_R \) is the respiration inhibition constant, describing the decrease of respiration in light.

Most of the model parameters are constant, except \( K \) and \( K_R \) for which independent oxygen or fluorescence measurements are used for proposing values, which are strain dependent.

The two-flux model can then be used to model the irradiance \( G(z) \) and the following formulation of irradiance distribution can be employed [30]:

\[
G(z) = \frac{2q_0(1 + \alpha)z^2 e^{-\delta(z)} - (1 - \alpha)z^2 e^{-\delta(L-z)}}{(1 + \alpha)z^2 e^{-\delta(L-z)} - (1 - \alpha)z^2 e^{-\deltaL}} \tag{5}
\]

where \( \delta = X\sqrt{E_o(E_o+2bE_o)} \) is the two-flux extinction coefficient, and \( \alpha = \sqrt{(E_o)/(E_o+2bE_o)} \) the linear scattering modulus. \( b \) is the backward scattering fraction (dimensionless). \( E_o \), the mass scattering coefficients (m\(^2\)/kg\(^{-1}\)), and \( E_a \) are chosen as a function of \( q_0 \), the incident light intensity. These optical properties are strain dependent and vary with growth conditions. Empirical formulӕ deduced from experiments in a wide range of incident light conditions [29] have been used here (see Table 1).

### Table 1

**Model parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L )</td>
<td>0.05</td>
<td>m</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.08</td>
<td>-</td>
</tr>
<tr>
<td>( E_o )</td>
<td>(-28 + \log(q_0) + 337)</td>
<td>m(^2)/kg</td>
</tr>
<tr>
<td>( E_a )</td>
<td>28.9 + \log(q_0) + 708</td>
<td>m(^2)/kg</td>
</tr>
<tr>
<td>( K )</td>
<td>120</td>
<td>( \mu )mol/m(^3)/h</td>
</tr>
<tr>
<td>( \Phi )</td>
<td>1.12 (10^{-7})</td>
<td>mol/( \mu )mol</td>
</tr>
<tr>
<td>( J_{NADH_2}/v_{NADH_2-O_2} )</td>
<td>3.19 (10^{-4})</td>
<td>-</td>
</tr>
<tr>
<td>( \varepsilon_a )</td>
<td>1.183</td>
<td>-</td>
</tr>
<tr>
<td>( M_s )</td>
<td>24 (10^{-1})</td>
<td>kgC/mol</td>
</tr>
<tr>
<td>( K_R )</td>
<td>6</td>
<td>( \mu )mol/m(^3)/s</td>
</tr>
<tr>
<td>( \rho_m )</td>
<td>0.8</td>
<td>-</td>
</tr>
</tbody>
</table>

### III. CONTROL STRATEGIES

#### A. Problem formulation

The aim is to get high cultivation in PBR, by improving the production and maintaining the quality of the product. One way to achieve this goal consists in regulating the biomass concentration at a given setpoint that leads to high production of the PBR. In the case of continuous operation mode, at constant incident light, the biomass concentration can be controlled hydraulically through the dilution rate \( D \) in open or closed-loop aiming maximum productivity, while avoiding washout, i.e., an unstable equilibrium point...
corresponding to the disappearance of the microorganisms from the cultivation system, where \( X=0 \). The photobioreactor can be operated at various concentrations of biomass in accordance with the selected working protocol. In this study, the regulation of the biomass concentration in a continuous PBR is considered, based on the so-called turbidostat protocol. The biomass concentration is assumed to be measured online via a turbidity sensor. It can also be determined from oxygen release measurements.

B. Setpoint determination

For constant incident light, the operating point (in terms of biomass concentration and dilution rate) can be determined so that the biomass productivity is maximized. The latter is defined as the product of steady state values of biomass concentration times dilution rate. It is equal to \( X \times D \) at the equilibrium. Experimental protocol can be defined to determine the optimal setpoint as a function of the applied incident light intensity. Consequently, the result can be illustrated as shown by Fig. 1 for 100 incident light intensity. Consequently, the result can be illustrated as shown by Fig. 1 for 100

\[
\hat{r}_X(X) = \frac{\mu_0}{K_I + \frac{\bar{G}}{G}} - \mu_r X
\]

(10)

where \( \mu_0 \) is related to the maximal specific growth rate, \( \mu_r \) represents the respiration rate, and \( K_I \) (resp. \( K_{II} \)) is the limitation (resp. inhibition) constant [20]. The mean incident light intensity \( G \) is given, with the same notations as previously, by:

\[
G = \frac{1}{L} \int_0^L q_0 \exp \left[ -\frac{1}{2\alpha} \hat{E}_r X \right] dz
\]

(11)

The parameters of the simplified model were determined from the same data as those in (2)-(5). They are given by Table II [33].

**TABLE II**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_0 )</td>
<td>0.14</td>
<td>( \mu )</td>
</tr>
<tr>
<td>( \mu_r )</td>
<td>0.013</td>
<td>h⁻¹</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.71</td>
<td>h⁻¹</td>
</tr>
<tr>
<td>( E_r )</td>
<td>15.1</td>
<td>m²/kg</td>
</tr>
<tr>
<td>( K_I )</td>
<td>120</td>
<td>( \mu ) mol/m²/s</td>
</tr>
<tr>
<td>( K_{II} )</td>
<td>500</td>
<td>( \mu ) mol/m²/s</td>
</tr>
</tbody>
</table>

D. Model-free control and intelligent controllers

1) The ultra-local model: For simplicity’s sake, let us restrict ourselves to SISO systems. The unknown global description of the plant is replaced by the ultra-local model:

\[
y = \hat{y} + au
\]

(12)

where:

- the control and output variables are \( u \) and \( y \),
- the derivation order of \( y \) is 1 like in most concrete situations,
- \( a \in \mathbb{R} \) is chosen by the practitioner such that \( au \) and \( \dot{y} \) are of the same magnitude.

The following comments might be useful:

- Equation (12) is only valid during a short time lapse. It must be continuously updated,
- \( \hat{y} \) is estimated via the knowledge of the control and output variables \( u \) and \( y \),
- \( \hat{y} \) subsumes not only the unknown structure of the system, which most of the time will be nonlinear, but also of any disturbance.

1See [22] for more details, and [23] for a hardware implementation.
Combining (12) and (13) yields:

\[ e = y - y_r \]

where:
- \( e \) is the tracking error,
- \( K_P \) is a usual tuning gain.

Combining (12) and (13) yields:

\[ \dot{e} + K_P e = 0 \]

where \( \bar{y} \) does not appear anymore. The tuning of \( K_P \) is therefore quite straightforward. This is a major benefit when compared to the tuning of “classic” PIDs (see, e.g., [34], and the references therein). Some more comments may be useful:

- See [35] for a connection with stability margins.
- Equations (13) and (14) render pointless any formal checking of the closed-loop system behavior (compare, e.g., with [36]).

3) Estimation of \( \bar{y} \): Assume that \( \bar{y} \) in (12) is “well” approximated by a piecewise constant function \( \bar{y}_{est} \).

1) Rewrite (12) by using the well-known notations from operational calculus:

\[ sY = \frac{\Phi}{s} + aU + y(0) \]

where \( \Phi \) is a constant. We get rid of the initial condition \( y(0) \) by multiplying both sides on the left by \( \frac{d}{dt} \):

\[ Y + \frac{dY}{ds} = -\frac{\Phi}{s^2} + a \frac{dU}{ds} \]

Noise attenuation is achieved by multiplying both sides on the left by \( s^{-2} \). It yields in the time domain the realtime estimate, thanks to the equivalence between \( \frac{d}{dt} \) and the multiplication by \( -1 \).

\[ \bar{y}_{est}(t) = \frac{6}{\tau} \int_{t-\tau}^{t} [(\tau - 2\sigma)\dot{y}(\sigma) + a\sigma(\tau - \sigma)u(\sigma)] d\sigma \]

where \( \tau > 0 \) might be quite small. This integral, which is a low pass filter, may of course be replaced in practice by a classic digital filter.

2) Close the loop with the iP (13). It yields:

\[ \bar{y}_{est}(t) = \int_{t-\tau}^{t} (\dot{y}_r - au - K_P e) d\sigma \]

IV. SIMULATION TESTS

A. Setpoint change with light

First, the two control laws are compared in the case of a piecewise-constant light intensity with the following profile:

\[ q_0(t) = \begin{cases} 600 \mu \text{mol m}^{-2} \text{s}^{-1} & \text{for } 0 < t \leq 30h \\ 100 \mu \text{mol m}^{-2} \text{s}^{-1} & \text{otherwise} \end{cases} \]

(15)

Fig. 1. Setpoint of biomass concentration and corresponding productivity and dilution rate versus constant applied incident light intensity.

Fig. 2. Block diagram of the control strategy.

1. Rewrite (12) by using the well-known notations from operational calculus:

\[ sY = \frac{\Phi}{s} + aU + y(0) \]

where \( \Phi \) is a constant. We get rid of the initial condition \( y(0) \) by multiplying both sides on the left by \( \frac{d}{dt} \):

\[ Y + \frac{dY}{ds} = -\frac{\Phi}{s^2} + a \frac{dU}{ds} \]

Noise attenuation is achieved by multiplying both sides on the left by \( s^{-2} \). It yields in the time domain the realtime estimate, thanks to the equivalence between \( \frac{d}{dt} \) and the multiplication by \( -1 \).

\[ \bar{y}_{est}(t) = \frac{6}{\tau} \int_{t-\tau}^{t} [(\tau - 2\sigma)\dot{y}(\sigma) + a\sigma(\tau - \sigma)u(\sigma)] d\sigma \]

where \( \tau > 0 \) might be quite small. This integral, which is a low pass filter, may of course be replaced in practice by a classic digital filter.

2) Close the loop with the iP (13). It yields:

\[ \bar{y}_{est}(t) = \int_{t-\tau}^{t} (\dot{y}_r - au - K_P e) d\sigma \]

Fig. 1. Setpoint of biomass concentration and corresponding productivity and dilution rate versus constant applied incident light intensity.
control is canceled, i.e., the system operates in the batch mode, so that the biomass concentration reaches almost the reference value. Then, after about 10 hours, the control input switches to a continuous mode and a dilution is applied so that the output is maintained at the desired value. At time $t = 30$ hours onwards, the reference value decreases to 0.17 kg/m$^3$. Consequently, the controllers apply a higher dilution rate to dilute the culture and attain the new reference value (after about 5 h). Then, the dilution rate reaches a constant value that maintains the output at this new reference.

It can be noticed that the model-based controller is sensitive to the value of the parameter $\mu_0$. Indeed, the reference tracking presents an offset in this case. The response with the model-free controller on the other hand is offset-free. Its robustness is highlighted.

B. Light change with constant setpoint

The controllers are now compared in the case of a constant biomass concentration setpoint $y_r = 0.175$ kg/m$^3$, with a time-varying incident light intensity. The latter is assumed to follow a profile depicted in Fig. 4, i.e., it is chosen in order to be similar to a day/night cycle of a solar light except the minimum level of incident light, which has been set to 100 $\mu$mol/m$^2$/s, here. The controllers tuning parameters and the simulation conditions are similar to those considered in Section IV-A. Simulation results, illustrated by Fig. 5, show that despite the light variation, considered here as a disturbance, the two control laws maintain the output at its reference value. Indeed, the dilution rate is modified, according to the incident light profile. Nevertheless, the model-free controller performs better than the model-based one. The model-based controller is again sensitive to the uncertainty of $\mu_0$. Its drawback is here also underlined.

V. CONCLUSION

The model-free control strategy yields better performances than the model-based one, in terms of

- setpoint tracking and robustness with respect to model uncertainties,
- no modeling prerequisite, i.e.,
  - only the output variable needs to be known,
  - the estimation of the state variables becomes useless,
- easy and costless implementation.

An experimental set-up and the control of the specific growth rate for insuring a high level of biomass growth are now being developed. They will hopefully be presented soon. The above preliminary results should not only be confirmed but also amplified.

REFERENCES


