



**HAL**  
open science

# Active queue management for alleviating Internet congestion via a nonlinear differential equation with a variable delay

Hugues Mounier, Cédric Join, Emmanuel Delaleau, Michel Fliess

## ► To cite this version:

Hugues Mounier, Cédric Join, Emmanuel Delaleau, Michel Fliess. Active queue management for alleviating Internet congestion via a nonlinear differential equation with a variable delay. *Annual Reviews in Control*, 2023, 55, pp.61-69. 10.1016/j.arcontrol.2023.02.002 . hal-03998960

**HAL Id: hal-03998960**

**<https://polytechnique.hal.science/hal-03998960v1>**

Submitted on 21 Feb 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Active queue management  
for alleviating Internet congestion  
via a nonlinear differential equation  
with a variable delay

February 21, 2023

**Hugues Mounier<sup>1</sup>, Cédric Join<sup>2,5</sup>, Emmanuel Delaleau<sup>3</sup> and  
Michel Fliess<sup>4,5,\*</sup>**

<sup>1</sup> L2S (CNRS, UMR 8506), Université Paris-Saclay,  
Centrale Supélec, 3 rue Joliot-Curie, 91192 Gif-sur-Yvette, France  
`hugues.mounier@universite-paris-saclay.fr`

<sup>2</sup> CRAN (CNRS, UMR 7039), Université de Lorraine,  
Campus Aiguillettes, B.P. 70239, Vandœuvre-ls-Nancy, 54506, France  
`cedric.join@univ-lorraine.fr`, `cedric.join@alien-sas.com`

<sup>3</sup> ENI Brest, UMR CNRS 6027, IRDL, F-29 200, France  
`emmanuel.delaleau@enib.fr`

<sup>4</sup> LIX (CNRS, UMR 7161), école polytechnique, 91128 Palaiseau, France  
`michel.fliess@polytechnique.edu`, `michel.fliess@swissknife.tech`,  
`michel.fliess@alien-sas.com`

<sup>5</sup> A.L.I.E.N., 7 rue Maurice Barrès, 54330 Vézelize, France

\* Corresponding author.

## Abstract

Active Queue Management (AQM) for mitigating Internet congestion has been addressed via various feedback control syntheses, especially P, PI, and PID regulators, by using a linear approximation where the “round trip time”, i.e., the delay, is assumed to be constant. This constraint is lifted here by using a nonlinear modeling with a variable delay, introduced more than 20 years ago. This delay, intimately linked to the congestion phenomenon, may be viewed as a “flat output.” All other system variables, especially the control variable, i.e., the packet loss ratio, are expressed as a function of the delay and its derivatives: they are frozen if the delay is kept constant. This flatness-like property, which demonstrates the mathematical discrepancy of the linear approximation adopted until today, yields also our control strategy in two steps: Firstly, designing an open-loop control, thanks to straightforward flatness-based control techniques, and secondly, closing the loop via Model-Free Control (MFC) in order to take into account severe model mismatches, like, here, the number of TCP sessions. Several convincing computer simulations, which are easily implementable, are presented and discussed.

## Highlights:

- In order to mitigate Internet congestion, this work is among the first ones to use a 20 years old modeling via a nonlinear differential equation with a variable delay, where a new flatness-like property is encountered: the delay is a flat output. Combining flatness-based open-loop control and closed-loop control via the intelligent proportional controller deduced from model-free control yields easily implementable and convincing computer experiments which display a remarkable robustness with respect to large uncertainties on the number of TCP connections.
- The above nonlinear modeling has mainly been employed until today to derive time-invariant linear delay approximate systems, which are quite popular, not only for investigating control-theoretic questions but also for computer experiments. The flatness-like property of the nonlinear model shows that freezing the delay implies that all other system variables, including the control one, are kept constant. The validity of the linear approximations is therefore questioned.

**Keywords:** Internet congestion, active queue management, flatness-based control, model-free control, intelligent proportional control, delay, nonstandard analysis, time series, prediction.

# 1 Introduction

In order to alleviate Internet congestion, an *active queue management* is a dropping packets policy inside a router buffer yielding a corresponding queue length management (see, e.g., [Adams(2013)], [Varma(2015)], [Grazia et al.(2017)], [Hotchi(2021)] for surveys and comparisons). It is often related to various control techniques and should perhaps be viewed, according to [Varma(2015)], as “the largest human-made feedback-controlled system in the world.” A modeling of the most popular *transmission control protocol* (*TCP*) has been derived more than twenty years ago in [Misra et al.(2000)] and [Hollot et al.(2002)] via some relationship with fluid mechanics. It is a nonlinear system of differential equations with a time-dependent delay, where the control variable is the packet loss ratio. Although this work is much cited, it seems, to the best of our knowledge, that almost only linear approximations with constant delays have been exploited to propose various applicable AQM techniques (see, however, [Barbera et al.(2010)], Belamfedel Alaoui et al.(2018), Li and Peng(2022)). Let us restrict our short review to a few examples where this approximation has been employed:

- The familiar *random early detection* (*RED*) algorithm, which was invented by [Floyd and Jacobson(1993)], has been commented by [Hollot et al.(2001)], [Ryu et al.(2005)].
- The well-known *proportional-integral enhanced* (*PIE*) controller is introduced by [Pan et al.(2013)].
- New algorithms are initiated by [Bisoy and Pattnaik(2017)], [Hotchi et al.(2020)], [Bisoy et al.(2021)], [Hotchi(2021)], [Hotchi and Kubo(2022)].

Our work relies on a remarkable attribute of the above mentioned nonlinear system: it should be called *flat* in a more or less analogous sense of [Fliess et al.(1995), Fliess et al.(1999)]. The delay is a *flat output*, the queue length another one. This means that the knowledge of the delay time variation, or of the queue length, determines all the other system variables, including the control one.

In particular, freezing the delay implies at once that all the other system variables are constant. This property thus questions the frequent use that seemed until today so self-evident, both for simulation and control purposes, of the linear approximations, where the delay is assumed to be constant and

not the other system variables (see, e.g., [Alli-Oke(2022)], and references therein).

This paper shows that appropriate control-theoretic tools do exist for handling the nonlinear modeling:

- Open-loop control strategies are deduced at once by exploiting this flatness-like property of the nonlinear modeling. We here choose to regulate the delay:
  - It is obviously related to congestion.
  - *Controlling queue Delay (CoDel)* ([Nichols and Jacobson(2012)]), which has become a popular setting, also puts delay control on the forefront but via a completely different viewpoint.
- In order to counteract the unavoidable model mismatches (see, e.g., [Xu et al.(2015)] for a summary of the shortcomings of the above nonlinear modeling) and disturbances, we follow [Villagra and Herrero-Perez(2012)], [Fliess et al.(2021)], [Join et al.(2022a)] by closing the loop via *model-free* control in the sense of [Fliess and Join(2013), Fliess and Join(2022)]. The presence of a delay requires a predictor which cannot, here, be the celebrated Smith’s predictor ([Smith(1957)]), because the latter is model-based (see, e.g., [Deng et al.(2022)] for a recent survey). We thus adapt here the viewpoint of [Hamiche et al.(2019)] for studying *supply chain management* (see also [Join et al.(2022b)]). This is achieved by removing the unpredictable *quick fluctuations* via a theorem due to [Cartier and Perrin(1995)] which is expressed in the language of *non-standard analysis*. Note that this result has led to a new understanding of time series (see, e.g., [Fliess et al.(2018)], and references therein).

Our paper is organized as follows. After showing that the delay or the queue length can be viewed as a flat output, Section 2 explains the inherent weakness of linear approximations with a constant delay. Section 3 recalls the basic facts of model-free control, which has already been used successfully many times. In order to take the delay into account in the model-free approach, Section 4 exploits techniques stemming from nonstandard analysis. Numerical simulations are presented in Section 5: they show that the model mismatch on the number of TCP sessions is well compensated by the closed-loop control without any clear-cut superiority of the techniques developed in Section 4. Various questions are raised in Section 6.

## 2 Some consequences of the nonlinear modeling

### 2.1 Flatness

The nonlinear TCP/AQM network model ([Misra et al.(2000), Hollot et al.(2002)]) reads

$$\dot{W}(t) = \frac{1}{R(t)} - \frac{W(t)W(t-R(t))}{2[R(t)-R(t)]}u(t-R(t)) \quad (1a)$$

$$\dot{Q}(t) = \frac{W(t)}{R(t)}N(t) - C(t) \quad (1b)$$

where

- $W(t) > 0$  is the length of the TCP window;
- $R(t) > 0$  is the *round trip time (RTT)* which appears as a time-dependent delay in Equation (1);
- $Q(t) > 0$  is the queue length;
- the control variable is the dropping packet policy  $u(t)$ ,  $0 \leq u(t) \leq 1$ : it is called the *packet loss ratio* ([Alli-Oke(2022)]), or, as often in the literature, the *packet drop probability*;
- $C(t) > 0$  is the bottleneck link capacity;
- $N(t) > 0$  is the number of TCP sessions. It plays the role of external disturbance.

In practice  $C(t)$  and  $N(t)$  are piecewise constant. Thus  $\dot{C}(t) = \dot{N}(t) = 0$ , with the exception of a finite number of points on any finite-time interval. The RTT  $R(t)$  and the queue length  $Q(t)$  are related by a simple affine relation

$$R(t) = T + \frac{Q(t)}{C(t)} \quad (2)$$

where  $T$  is the round trip propagation time. Assume, in the following computations,  $\dot{C}(t) = \dot{N}(t) = 0$  on some time interval, Set  $C(t) = C$ ,  $N(t) = N$ ,

where  $C$  and  $N$  are constant. Then Equation (2) yields  $\dot{Q}(t) = C\dot{R}(t)$ . Equation (1b) becomes

$$\dot{R}(t) = \frac{NW(t)}{CR(t)} - 1 \quad (3)$$

It yields

$$W(t) = \frac{CR(t)(\dot{R}(t) + 1)}{N} \quad (4)$$

Thus Equations (4) and (1a) show that  $W(t)$  and  $u(t - R(t))$  depend on  $R(t)$  and its first and second order derivatives. In other words, we may call System (1) *flat* and the RTT  $R(t)$  a *flat output* (compare with [Mounier et al.(2003)]). Equation (2) shows that the queue length  $Q(t)$  is another flat output.

**Remark 1** *Classic flatness has been formally defined in [Fliess et al.(1995)] via differential algebra and in [Fliess et al.(1999)] via differential geometry of infinite jets and prolongations. Combining differential and difference algebras (see, e.g., [Cohn(1970)]) permits a precise definition of flatness for nonlinear systems with constant delays ([Mounier and Rudolph(1998), Mounier and Rudolph(2008)]). Such a setting does not however work with a variable delay such as  $R(t)$ , since the time derivation  $\frac{d}{dt}$  and the time shift  $t \mapsto t - R(t)$  with a time-varying quantity do not commute. Before developing an adequate general mathematical formalism for our example, it may be wise to wait for other concrete case-studies. It would open a path to a new understanding of nonlinear systems with variable delays.*

## 2.2 Critical appraisal of the linear approximation

See [Alli-Oke(2022)] for a nice survey on linear approximations. Let  $u_0, Q_0, W_0, R_0$  be the numerical values of  $u(t), Q(t), W(t), R(t)$  at an operating, or equilibrium, point. Contrarily to the variables  $u, Q, W$ , the delay  $R$  is kept frozen at the value  $R_0$ : the delay in the linear approximation is constant. It is obvious that such an assumption contradicts Section 2.1, where Equations (2), (4) and (1a) show that  $Q(t), W(t)$  and  $u(t)$  become constant when  $R(t)$  is constant. This fact is casting some doubt not only about AQM via such approximations, but also on the computer simulations, which rely on it (see [Alli-Oke(2021), Alli-Oke(2022)], and the references therein).

**Remark 2** *Define the control variable  $\delta u(t) = u(t) - u_0$  and the output variable  $\delta Q(t) = Q(t) - Q_0$ . They are often related in the literature (see,*



e.g., [Alli-Oke(2022)]) by the time-invariant linear delay system defined by the transfer function

$$-\frac{(2N\frac{W_0}{2})^3 e^{-R_0 s}}{(R_0 s + 1)(\frac{W_0 R_0}{2} s + 1)}$$

where the number  $N$  of sessions is assumed to be constant. A system defined by such a transfer function is sometimes called quasi-finite ([Fliess et al.(2002)]). The output  $\delta Q$  is said to be flat, or basic ([Fliess et al.(2002)]).

### 3 Closed-loop control via model-free control without delay: a short review

#### 3.1 Ultra-local model

Consider a single-input single-output (SISO) nonlinear system ( $\Sigma$ ). Denote by  $\mathbf{u}(t)$  (resp.  $\boldsymbol{\eta}(t)$ ) the control (resp. output) variable. It has been demonstrated ([Fliess and Join(2013)]) via elementary techniques from functional analysis and differential algebra that the often poorly known modeling of ( $\Sigma$ ) may be replaced, if some quite weak assumptions are satisfied, by an *ultra-local model*:

$$\boldsymbol{\eta}^{(\nu)} = F + \alpha \mathbf{u} \tag{5}$$

where  $\alpha \in \mathbb{R}$  is chosen by the practitioner such that  $\alpha \mathbf{u}$  and  $\boldsymbol{\eta}^{(\nu)}$  are of the same order of magnitude: it does not need to be precisely known. Numerous successful applications (see, e.g., references in [Fliess and Join(2013), Fliess and Join(2022)]) have shown that  $\nu = 1$  in Equation (5) yields most often a convenient ultra-local model:

$$\dot{\boldsymbol{\eta}} = F + \alpha \mathbf{u} \tag{6}$$

The following comments are useful:

- Equation (6) is only valid during a short time lapse: it must be continuously updated.
- $F$  is *data-driven*, i.e., it is estimated via the knowledge  $\mathbf{u}$  and  $\boldsymbol{\eta}$  ([Fliess and Join(2013)]):

$$F_{\text{est}}(t) = -\frac{6}{\tau^3} \int_{t-\tau}^t [(\tau - 2\sigma)\boldsymbol{\eta}(\sigma) + \alpha\sigma(\tau - \sigma)\mathbf{u}(\sigma)] d\sigma \tag{7}$$

The quantity  $\tau > 0$  may be chosen to be quite “small.” The above integral, which is a low pass filter, may, in practice, be replaced by a classic digital filter.

- $F$  subsumes not only the unknown structure of the system, which most of the time is nonlinear, but also any external disturbance.

### 3.2 Intelligent controllers and local stability

The loop is closed with the following *intelligent proportional controller* ([Fliess and Join(2013)]), or *iP*,

$$\mathbf{u} = -\frac{F_{\text{est}} - \dot{\boldsymbol{\eta}}^* + K_P e}{\alpha} \quad (8)$$

where:

- $\boldsymbol{\eta}^*$  is the reference trajectory of the output,
- $e = \boldsymbol{\eta} - \boldsymbol{\eta}^*$  is the tracking error,
- $K_P \in \mathbb{R}$  is a tuning gain.

Combining Equations (6) and (8) yields  $\dot{e} + K_P e = F - F_{\text{est}}$ . If the estimate  $F_{\text{est}}$  is “good,” i.e., if  $F - F_{\text{est}} \approx 0$ , then  $\lim_{t \rightarrow +\infty} e(t) \approx 0$ , if, and only if,  $K_P > 0$ .

## 4 Closed-loop control via model-free control with delay

### 4.1 Ultra-local model with delay

Set  $v(t) = u(t - R(t))$  in Equation (1):

$$\begin{aligned} \dot{W}(t) &= \frac{1}{R(t)} - \frac{W(t)W(t - R(t))}{2[R(t) - R(t)]} v(t) \\ \dot{Q}(t) &= \frac{W(t)}{R(t)} N(t) - C(t) \end{aligned}$$

This trivial change of variable shows that the techniques from [Fliess and Join(2013)] remain valid for introducing the ultra-local model with a time-varying delay

$$\dot{\boldsymbol{\eta}}(t) = \boldsymbol{\mathfrak{F}} + \alpha \mathbf{u}(t - R(t)) \quad (10)$$

where  $\mathfrak{F}$  plays the same rôle as  $F$  in Equation (6). Equation (7) becomes

$$\mathfrak{F}_{\text{est}}(t) = -\frac{6}{\tau^3} \int_{t-\tau}^t [(\tau - 2\sigma)\eta(\sigma) + \alpha\sigma(\tau - \sigma)\mathbf{u}(\sigma - R(\sigma))] d\sigma \quad (11)$$

## 4.2 Prediction via time series

### 4.2.1 Time series and the Cartier-Perrin theorem

Consider the time interval  $[0, 1] \subset \mathbb{R}$ . Introduce as often in *nonstandard analysis* (see [Robinson(1974)], [Diener and Diener(1995)], [Lobry and Sari(2008)]) the infinitesimal sampling of  $[0, 1]$ :  $\mathfrak{T} = \{0 = t_0 < t_1 < \dots < t_\nu = 1\}$  where  $t_{i+1} - t_i$ ,  $0 \leq i < \nu$ , is *infinitesimal*, *i.e.*, “very small”. A *time series*  $X(t)$  is a function  $X : \mathfrak{T} \rightarrow \mathbb{R}$ .

A time series  $\mathcal{X} : \mathfrak{T} \rightarrow \mathbb{R}$  is said to be *quickly fluctuating*, or *oscillating*, if the integral  $\int_A \mathcal{X} dm$  is infinitesimal, *i.e.*, very small, for any *appreciable* interval, *i.e.*, an interval which is neither “very small” nor “very large”.

According to a theorem due to [Cartier and Perrin(1995)], the following additive decomposition holds for any time series  $X$ , which satisfies a weak integrability condition,

$$X(t) = E(X)(t) + X_{\text{fluctuation}}(t) \quad (12)$$

where

- the *mean*, or *trend*,  $E(X)$  is “quite smooth”;
- $X_{\text{fluctuation}}$  is quickly fluctuating.

The decomposition (12) is unique up to an additive infinitesimal: It means that the two terms on the right handside of Equation (12) are unique up to a “very small” additive quantity.

### 4.2.2 Derivative estimate

Let us start with a polynomial time function of degree 1

$$p_1(\tau) = a_0 + a_1\tau$$

where  $\tau \geq 0$ ,  $a_0, a_1 \in \mathbb{R}$ . Operational calculus (see, *e.g.*, [Yosida(1984)]) with respect to the variable  $\tau$ , permits to express  $p_1$  as

$$P_1 = a_0/s + a_1/s^2$$

Multiply both sides by  $s^2$ :

$$s^2 P_1 = a_0 s + a_1 \quad (13)$$

Take the derivative of both sides with respect to  $s$ , which corresponds in the time domain to the multiplication by  $-\tau$ :

$$s^2 \frac{dP_1}{ds} + 2sP_1 = a_0 \quad (14)$$

The coefficients  $a_0, a_1$  are obtained via the triangular system of linear equations (13)-(14). We get rid of the time derivatives, *i.e.*, of  $sP_1$ ,  $s^2P_1$ , and  $s^2 \frac{dP_1}{ds}$ , by multiplying both sides of Equations (13)-(14) by  $s^{-n}$ ,  $n \geq 2$ . The corresponding iterated time integrals are low pass filters which attenuate the corrupting noises. A quite short time window is sufficient for obtaining accurate values of  $a_0, a_1$ .

**Remark 3** See [Mboup et al.(2009)] and [Othmane et al.(2021), Othmane et al.(2022)] for more details. Note also that estimating derivatives via integrals seems to have been first introduced by [Lanczos(1956)].

### 4.2.3 Prediction

Set the following forecast  $X_{\text{forecast}}(t + \Delta T)$ , where  $\Delta T > 0$  is not too “large”,

$$X_{\text{forecast}}(t + \Delta T) = E(X)(t) + \left[ \frac{dE(X)(t)}{dt} \right]_e \Delta T \quad (15)$$

where  $E(X)(t)$  and  $\left[ \frac{dE(X)(t)}{dt} \right]_e$  are estimated like  $a_0$  and  $a_1$  above. Let us stress that what we predict is the mean and not the quick fluctuations.

**Remark 4** The above construction is obviously reminiscent of the sliding window techniques in the applied literature on time series (see, e.g., [Mélard(2008)]).

Note that estimating  $a_0$  and  $a_1$  yields respectively the mean and the derivative.

#### 4.2.4 Local closed-loop stability

Equation (10) may be rewritten as

$$\dot{\mathfrak{h}}(t + S(t)) = \mathfrak{F}_{\text{forecast}}(t + S(t)) + \alpha \mathbf{u}(t) \quad (16)$$

where the advance  $S(t) > 0$  is defined by

$$S(t) = \min \{ \tau \mid \tau - R(\tau) = t \} - t \quad (17)$$

If  $R(t)$  is “slowly” varying, it is clear that  $R(t)$  and  $S(t)$  remain close. Evaluating  $S(t)$  however requires a prediction of  $R(t)$ . Replace therefore in Equation (17)  $R(t)$  by the reference trajectory  $R^*(t)$ . It yields

$$S^*(t) = \min \{ \tau \mid \tau - R^*(\tau) = t \} - t \quad (18)$$

Equation (8) then becomes

$$\mathbf{u}(t) = - \frac{\mathfrak{F}_{\text{forecast}}(t + S^*(t)) - \dot{\mathfrak{h}}^*(t + S^*(t)) + K_P e(t + S^*(t))}{\alpha} \quad (19)$$

where:

- the forecast of  $\mathfrak{F}$  is obtained via Formulae (11) and (15);
- $e(t + S^*(t)) = \mathfrak{h}_{\text{forecast}}(t + S^*(t)) - \mathfrak{h}^*(t + S^*(t))$ , where  $\mathfrak{h}^*$  is the reference trajectory and  $e$  is the tracking error;
- $\mathfrak{h}_{\text{forecast}}(t + S^*(t)) = \mathfrak{z}(t + S^*(t))$  is obtained via the linear differential equation

$$\dot{\mathfrak{z}}(\tau) = \mathfrak{F}_{\text{forecast}}(\tau) + \alpha \mathbf{u}(\tau - R^*(\tau)) \quad t \leq \tau \leq t + S(t)$$

- $K_P \in \mathbb{R}$  is the tuning gain.

It yields

$$\dot{e}(t + S^*(t)) + K_P e(t + S^*(t)) = \mathfrak{F}(t + S^*(t)) - \mathfrak{F}_{\text{forecast}}(t + S^*(t)) \quad (20)$$

Local stability is ensured, i.e.,  $\lim_{t \rightarrow +\infty} \mathfrak{h}(t + S^*(t)) \approx \mathfrak{h}_{\text{forecast}}(t + S^*(t))$ , if

- $K_P > 0$ ,
- the forecast is “good,” i.e.,  $\mathfrak{F}(t + S^*(t)) - \mathfrak{F}_{\text{forecast}}(t + S^*(t)) \approx 0$ .

## 5 Computer simulations<sup>1</sup>

### 5.1 Various situations

Introduce the following control settings:

1. **Reference trajectory and nominal control:** The choice of a reference trajectory  $R^*(t)$  for the delay  $R(t)$  yields at once via Section 2.1 an open-loop nominal control  $u^*(t)$  for  $u(t)$ , i.e.,

$$u^*(t - R^*(t)) = 2 \left( \frac{1}{R^*(t)} - \dot{W}^*(t) \right) \left( \frac{R^*(t - R^*(t))}{W^*(t)W^*(t - R^*(t))} \right)$$

or

$$u^*(t) = 2 \left( \frac{1}{R^*(t + S^*(t))} - \dot{W}^*(t + S^*(t)) \right) \left( \frac{R^*(t)}{W^*(t + S^*(t))W^*(t)} \right)$$

and

$$W^*(t) = \frac{C(t)R^*(t)(\dot{R}^*(t) + 1)}{N_0}$$

where  $N_0 = 60$  and  $C = 3000$ .

2. **Open-loop control (OL):** Inject  $u^*$  in Equation (1) with  $N(t) = N_0$ .
3. **Closing the loop via an iP without delay (iP):** In Equations (6), (7), (8), set  $\eta(t) = e(t) = R(t) - R^*(t)$ ,  $\eta^*(t) = 0$ ,  $\mathbf{u}(t) = \Delta u(t) = u(t) - u^*(t)$ . The iP (8) becomes

$$\Delta u = -\frac{F_{\text{est}} + K_P e}{\alpha}$$

where  $\alpha = -1000$ ,  $K_P = 1$ . Consider the estimation of  $F_{\text{est}}$  via Formula (7) as a classic *finite impulse response (FIR)* (see, e.g., [Rabiner and Gold(1975)]). We thus apply a control  $u(t)$  of the form

$$u(t) = u^*(t) - \frac{F_{\text{est}} + K_P e}{\alpha}$$

where the first part incorporates our knowledge of the system, and the second one deals with perturbations, model imperfections and unknown dynamics. It is clear that the practical implementation, which has already been achieved successfully a number of times, is straightforward.

---

<sup>1</sup>Contact C. Join ([cedric.join@univ-lorraine.fr](mailto:cedric.join@univ-lorraine.fr)) for the simulation codes.

4. **Closing the loop via an iP with delay (iPWD):** In Equations (10), (11), (19), set as above  $\eta(t) = e(t) = R(t) - R^*(t)$ ,  $\eta^*(t) = 0$ ,  $u(t) = \Delta u(t) = u(t) - u^*(t)$ . The iP (19) becomes

$$\Delta u(t) = -\frac{\tilde{\mathfrak{F}}_{\text{forecast}}(t + S^*(t)) + K_P e(t + S^*(t))}{\alpha}$$

where, as above,  $\alpha = -10$ ,  $K_P = 1$ . The calculations related to predictions are detailed in [Fliess et al.(2018)]. The sequel is similar to the case without delay.

## 5.2 Scenarios

We illustrate our control laws through two different scenarios, where the mismatch is the number of TCP sessions. The first scenario corresponds to normal operation: small variation of R. The second scenario represents an exit from a congestion situation and corresponds to a large variation of R. The command is designed with a known value of the number of connections N but operates with a N constant piecewise: in our simulation N first goes from 60 to 70 then goes down to 50. The number of connection N plays the role of an external disturbance. It is moreover this effect that the open loop curves Figs.1 and 4 show where the trajectory deviates from the reference one when the number of connections changes.

1. Scenario 1:  $0.25s \leq R(t) \leq 0.3s$ ,  $50 \leq N(t) \leq 70$ .
2. Scenario 2:  $0.3s \leq R(t) \leq 0.7s$ ,  $50 \leq N(t) \leq 70$ .

Table 1: Simulations and Figures

Scenarios	OL	iP	iPWD
1	Figure 1	Figure 2	Figure 3
2	Figure 4	Figure 5	Figure 6

Those Figures tell us that:

- the mismatch is well compensated by iP with or without delay,
- iP with or without delay exhibit very similar behaviors.

Our results are especially well displayed in Figure 7. Indeed, we can see that the behavior of the tracking errors are totally comparable. In other words the iP with delay seems to be useless!

## 6 Conclusion

It has been shown that

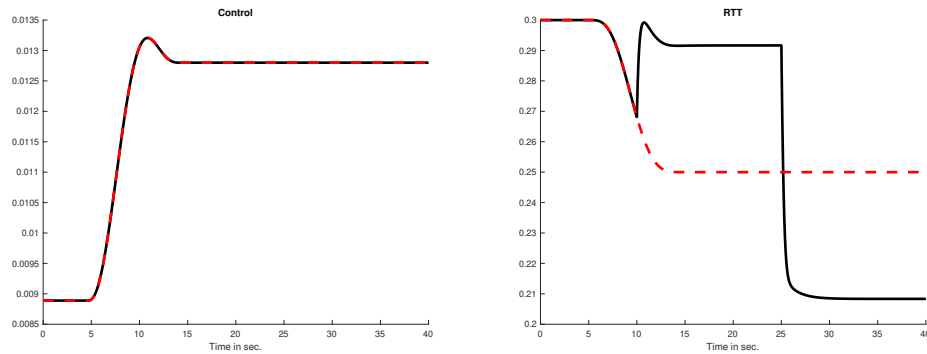
- the linear constant delay approximations, which also play a key rôle in computer simulations, contradict the more complete nonlinear modeling;
- control-theoretic tools are available for an active queue management via this nonlinear modeling.

Many points remain of course to be addressed:

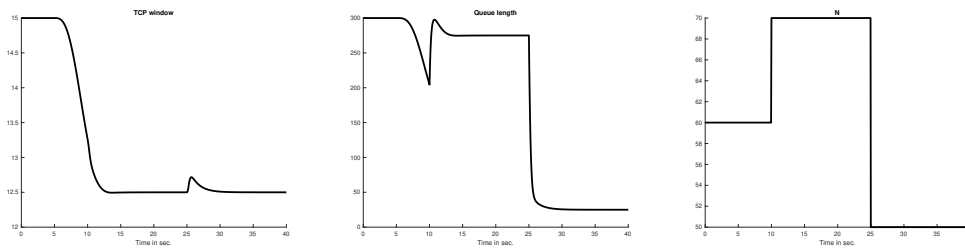
- Other mismatches and external disturbances ought to be examined: noisy measurements, abrupt changes of the round trip time, ... Would the *intelligent proportional-derivative controller (iPD)* advocated by [Fliess and Join(2022)] be helpful? See, e.g., [Sun et al.(2003)] and [Ryu et al.(2005)] for results with classic PD controllers.
- The simulations in Section 5.2 indicate the futility of an iP with delay in order to compensate a model mismatch. Without a precise mathematical analysis, it is not clear whether this property is always valid. Let us suggest nevertheless that the open-loop nonlinear control, where the delay is taken into account, is doing the job!
- The coefficient  $\alpha$  in Equations (6) and (10), which does not need to be determined precisely, is obtained via trials and errors. A more subtle estimation would be welcome. Let us also add that important variations of some quantities like the number  $N(t)$  of TCP connections might necessitate the introduction of a time-varying  $\alpha$  (see [Gédoin et al.(2011), Moreno-Gonzales et al.(2022)] for first results in other engineering domains).
- Many network simulation for investigating Internet congestion (see, e.g., [Riley and Henderson(2010)], [Alli-Oke(2021), Alli-Oke(2022)], and references therein) seem to have employed time-invariant linear delay



systems (see Section 2.2). It should therefore be most rewarding to develop and integrate the tools of this paper.



(a) Control (-) and nominal control (- -)    (b)  $R$  (-) and reference trajectory (- -)

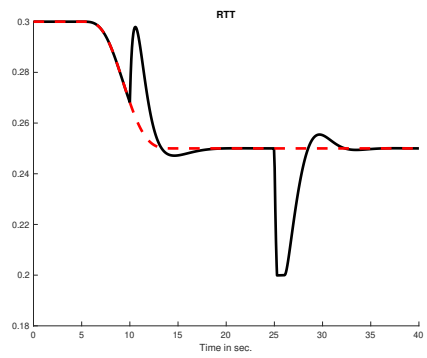
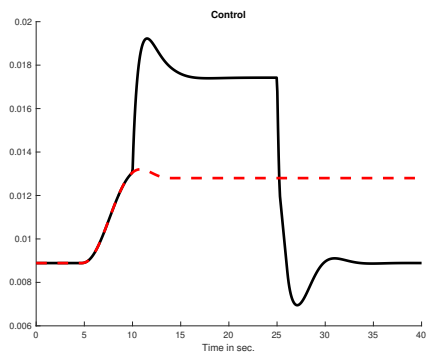


(c) TCP Window

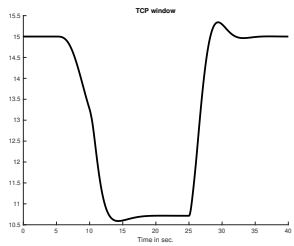
(d) Queue length

(e) Number of connections

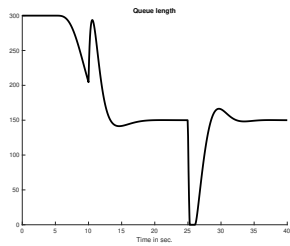
Figure 1: Scenario 1 – OL



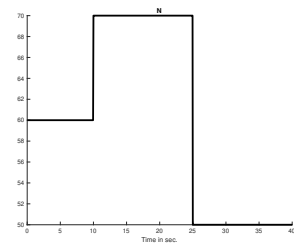
(a) Control (-) and nominal control (- -)    (b)  $R$  (-) and reference trajectory (- -)



(c) TCP Window

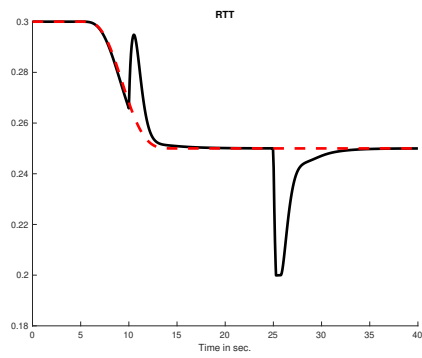
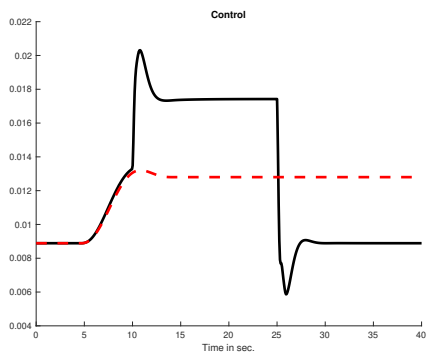


(d) Queue length

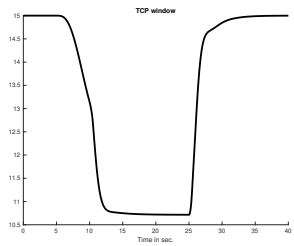


(e) Number of connections

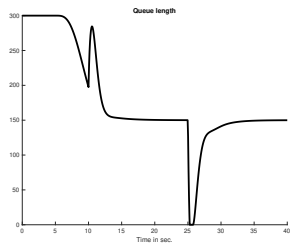
Figure 2: Scenario 1 – iP



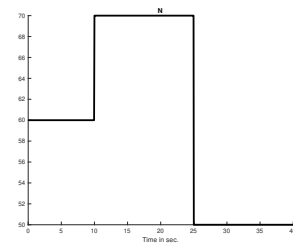
(a) Control (—) and nominal control (---)    (b)  $R$  (—) and reference trajectory (---)



(c) TCP Window

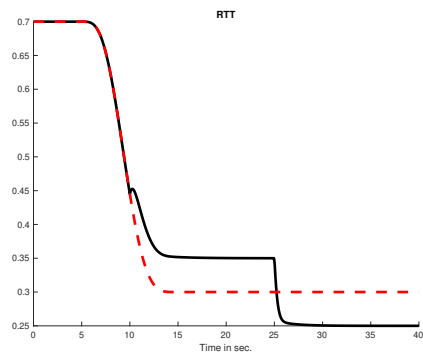
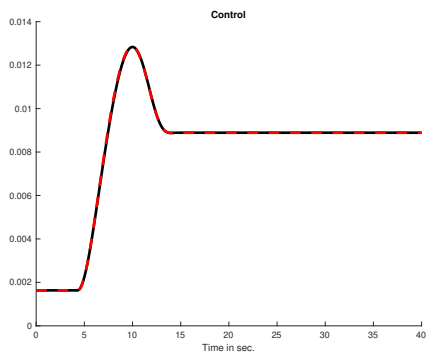


(d) Queue length

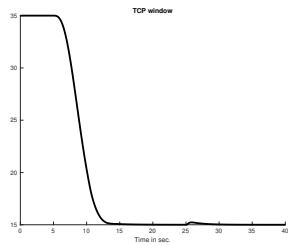


(e) Number of connections

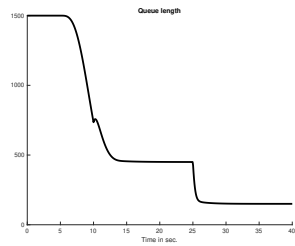
Figure 3: Scenario 1 – iPWD



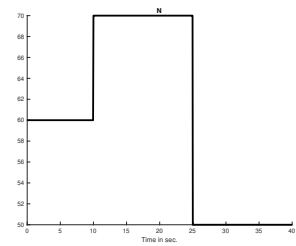
(a) Control (-) and nominal control (- -)    (b)  $R$  (-) and reference trajectory (- -)



(c) TCP Window

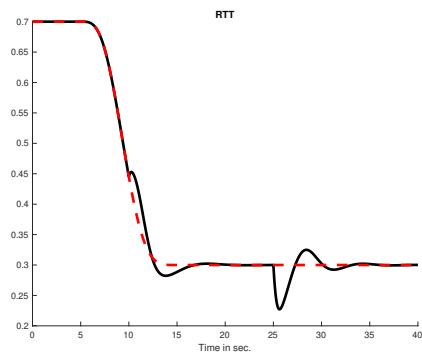
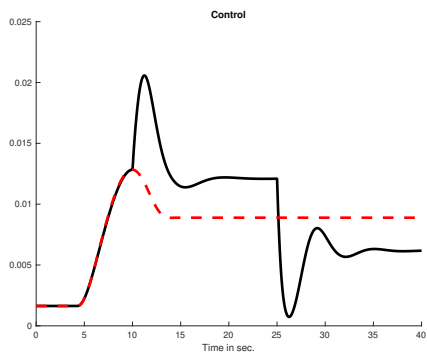


(d) Queue length

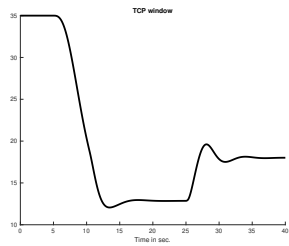


(e) Number of connections

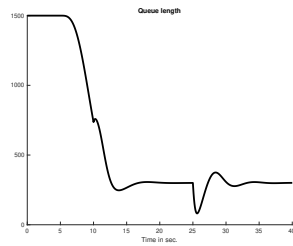
Figure 4: Scenario 2 – OL



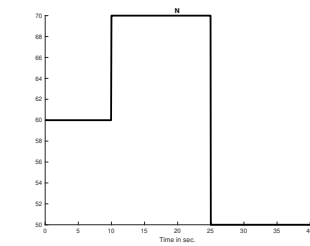
(a) Control (-) and nominal control (- -)    (b)  $R$  (-) and reference trajectory (- -)



(c) TCP Window

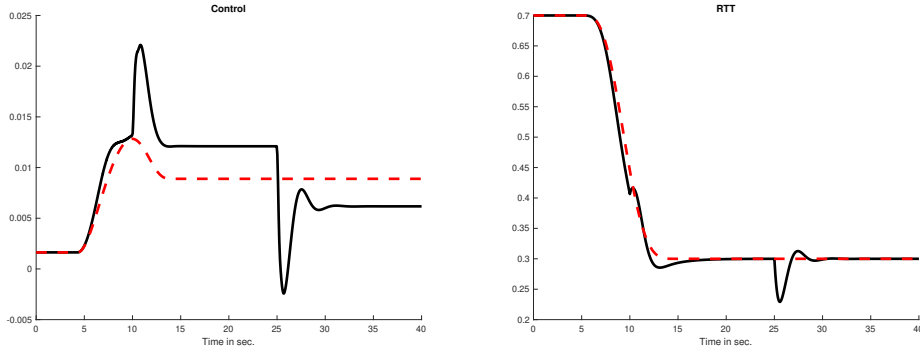


(d) Queue length

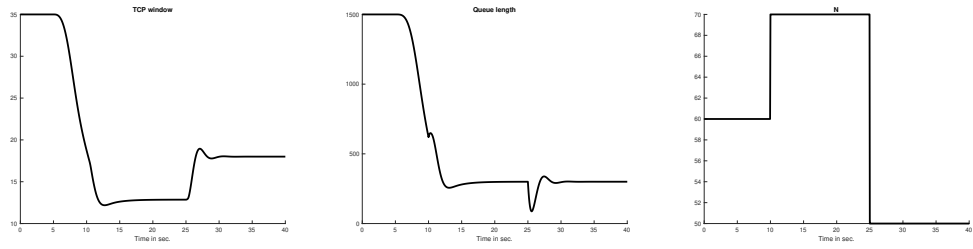


(e) Number of connections

Figure 5: Scenario 2 – iP

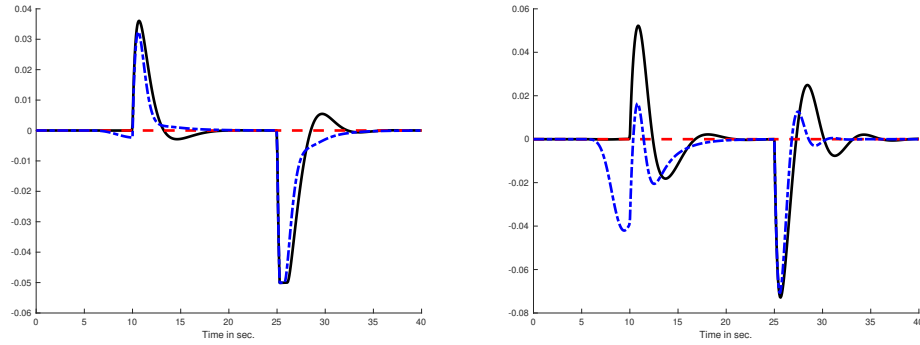


(a) Control (-) and nominal control (- -) (b)  $R$  (-) and reference trajectory (- -)



(c) TCP Window (d) Queue length (e) Number of connections

Figure 6: Scenario 2 – iPWD



(a) Scenario 1 iP (-), iPWD (- .) and zero line (- -) (b) Scenario 2 iP (-), iPWD (- .) and zero line (- -)

Figure 7: Tracking errors

## References

- [Adams(2013)] Adams, R., 2013. Active queue management: A survey. *IEEE Communications Surveys & Tutorials* 15, 1425–1476.
- [Alli-Oke(2021)] Alli-Oke R.O., 2021. Repository. <https://github.com/droa28/AQM-Simulations>
- [Alli-Oke(2022)] Alli-Oke, R.O., 2022. On the validity of numerical simulations for control-theoretic AQM schemes in computer networks. *Mathematics and Computers in Simulation* 193, 466–480.
- [Barbera et al.(2010)] Barbera M., Lombardo A., Panarello C., Schembra G., 2010. Queue stability analysis and performance evaluation of a TCP-compliant window management mechanism. *IEEE/ACM Transactions on Networking* 18, 1275–1288.
- [Belamfedel Alaoui et al.(2018)] Belamfedel Alaoui S., Tissir E.H., Chaibi N., 2018. Active queue management based feedback control for TCP with successive delays in single and multiple bottleneck topology. *Comput. Communicat.* 117, 58–70.
- [Bisoy and Pattnaik(2017)] Bisoy, S.K., Pattnaik, P.K., 2017. Design of feedback controller for TCP/AQM networks. *Engineering Science and Technology* 20, 116–132.
- [Bisoy et al.(2021)] Bisoy, S.K., Pattnaik, P.K., Sain, M., Jeong, D.U., 2021. A self-tuning congestion tracking control for TCP/AQM network for single and multiple bottleneck topology. *IEEE Access* 9, 27723–27735.
- [Cartier and Perrin(1995)] Cartier, P., Perrin, Y., 1995. Integration over finite sets. In Diener, F., Diener, M. (Eds.), *Nonstandard Analysis in Practice*. Springer, Berlin, Heidelberg, pp. 185–204.
- [Cohn(1970)] Cohn, R.M., 1970. A difference-differential basis theorem. *Canadian Journal of Mathematics* 22, 1224–1237. doi:10.4153/CJM-1970-141-3.
- [Deng et al.(2022)] Deng, Y., L echapp e, V., Moulay, E., Chen, Z., Liang, B., Plestan, F., Han, Q.L., 2022. Predictor-based control of time-delay

- systems: a survey. *International Journal of Systems Science* 53, 2496–2534.
- [Diener and Diener(1995)] Diener, F., Diener, M., 1995. Tutorial. In Diener, F., Diener, M. (Eds.), *Nonstandard Analysis in Practice*. Springer, Berlin, Heidelberg, pp. 1–21.
- [Fliess and Join(2013)] Fliess, M., Join, C., 2013. Model-free control. *International Journal of Control* 86, 2228–2252.
- [Fliess and Join(2022)] Fliess, M., Join, C., 2022. An alternative to proportional-integral and proportional-integral-derivative regulators: Intelligent proportional-derivative regulators. *International Journal of Robust and Nonlinear Control* 32, 9512–9524.
- [Fliess et al.(2021)] Fliess, M., Join, C., Moussa, K., Djouadi, S.M., Alsager, M.W., 2021. Toward simple in silico experiments for drugs administration in some cancer treatments. *IFAC-PapersOnLine* 54, 245–250.
- [Fliess et al.(2018)] Fliess, M., Join, C., Voyant, C., 2018. Prediction bands for solar energy: New short-term time series forecasting techniques. *Solar Energy* 166, 519–528.
- [Fliess et al.(1995)] Fliess, M., Lévine, J., Martin, P., Rouchon, P., 1995. Flatness and defect of non-linear systems: introductory theory and examples. *International Journal of Control* 61, 1327–1361.
- [Fliess et al.(1999)] Fliess, M., Lévine, J., Martin, P., Rouchon, P., 1999. A Lie-Bäcklund approach to equivalence and flatness of nonlinear systems. *IEEE Transactions on Automatic Control* 44, 922–937.
- [Fliess et al.(2002)] Fliess, M., Marquez, R., Mounier, H., 2002. An extension of predictive control, PID regulators and Smith predictors to some linear delay systems. *International Journal of Control* 75, 728–743.
- [Floyd and Jacobson(1993)] Floyd, S., Jacobson, V., 1993. Random early detection gateways for congestion avoidance. *IEEE/ACM Transactions on Networking* 1, 397–413.
- [Gédoin et al.(2011)] Gédoin P.-A., Delaleau E., Bourgeot J.-M., Join C., Arbab Chirani S., Calloch S., 2011. Experimental comparison of classical



PID and model-free control: Position control of a shape memory alloy active spring. *Control Engineering Practice* 19, 433–441.

- [Grazia et al.(2017) ] Grazia C.A., Patriciello N., Klapez M., M. Casoni M., 2017. A cross-comparison between TCP and AQM algorithms: Which is the best couple for congestion control?. 2017 14th International Conference on Telecommunications (ConTEL), Zagreb, pp. 75–82.
- [Hamiche et al.(2019)] Hamiche, K., Fliess, M., Join, C., Abouassa, H., 2019. Bullwhip effect attenuation in supply chain management via control-theoretic tools and short-term forecasts: A preliminary study with an application to perishable inventories, in: 2019 6th International Conference on Control, Decision and Information Technologies (CoDIT), Paris, pp. 1492–1497.
- [Hollot et al.(2001)] Hollot, C., Misra, V., Towsley, D., Gong, W.B., 2001. A control theoretic analysis of RED. In: *Proc. IEEE INFOCOM 2001*, pp. 1510–1519, vol. 3.
- [Hollot et al.(2002)] Hollot, C., Misra, V., Towsley, D., Gong, W., 2002. Analysis and design of controllers for AQM routers supporting TCP flows. *IEEE Transactions on Automatic Control* 47, 945–959.
- [Hotchi(2021)] Hotchi, R.A., 2021. Active queue management based on control-theoretic approaches for diversified communication services. PhD Thesis, Keio University, Tokyo.
- [Hotchi et al.(2020)] Hotchi, R., Chibana, H., Iwai, T., Kubo, R., 2020. Active queue management supporting TCP flows using disturbance observer and smith predictor. *IEEE Access* 8, 173401–173413.
- [Hotchi and Kubo(2022)] Hotchi, R., Kubo, R., 2022. Quality of service aware adaptive target queue length generation for active queue management. *IET Control Theory & Applications* 16, 398–413.
- [Join et al.(2022a)] Join, C., d’Onofrio, A., Fliess, M., 2022a. Toward more realistic social distancing policies via advanced feedback control. *Automation* 3, 286–301.

- [Join et al.(2022b)] Join, C., Mounier, H., Delaleau, E., Fliess, M., 2022b. Active queue management: First steps toward a new control-theoretic viewpoint. International Conference on Systems and Control (ICSC'2022), Marseille, 448–453.
- [Lanczos(1956)] Lanczos, C., 1956. Applied Analysis. Prentice-Hall, Englewood Cliffs.
- [Li and Peng(2022)] Li, C., Deng, D., 2022. Uniform stability of nonlinear systems with state-dependent delay. Automatica 137, 110098.
- [Lobry and Sari(2008)] Lobry, C., Sari, T., 2008. Non-standard analysis and representation of reality. International Journal of Control 81, 519–536.
- [Mboup et al.(2009)] Mboup, M., Join, C., Fliess, M., 2009. Numerical differentiation with annihilators in noisy environment. Numerical Algorithms 50, 439–467.
- [Mélard(2008)] Mélard, G., 2008. Méthodes de prévision à court terme (2<sup>e</sup> éd.). Ellipses & Presses Universitaires de Bruxelles, Paris & Bruxelles.
- [Misra et al.(2000)] Misra, V., Gong, W.B., Towsley, D., 2000. Fluid-based analysis of a network of AQM routers supporting TCP flows with an application to RED. SIGCOMM Comput. Commun. Rev. , 151–160.
- [Moreno-Gonzales et al.(2022)] Moreno-Gonzalez M., Artuñedo A., Villagra J., Join C., Fliess M., 2022. Speed-adaptive model-free lateral control for automated cars. IFAC-PapersOnLine 55, 84–89.
- [Mounier et al.(2003)] Mounier, H., Bastin, G., Vèque, V., Zitoune, L., 2003. Round trip time TCP tracking: a first step towards QoS pricing. International Journal of Systems Science 34, 607–614.
- [Mounier and Rudolph(1998)] Mounier, H., Rudolph, J., 1998. Flatness-based control of nonlinear delay systems: A chemical reactor example. International Journal of Control 71, 871–890.
- [Mounier and Rudolph(2008)] Mounier, H., Rudolph, J., 2008. Flatness and quasi-static state feedback in non-linear delay systems. International Journal of Control 81, 445–456.

- [Nichols and Jacobson(2012)] Nichols, K., Jacobson, V., 2012. Controlling queue delay: A modern AQM is just one piece of the solution to bufferbloat. *ACMQueue* 5, 1–20.
- [Othmane et al.(2022)] Othmane, A., Kiltz, L., Rudolph, J., 2022. Survey on algebraic numerical differentiation: historical developments, parametrization, examples, and applications. *International Journal of Systems Science* 53, 1848–1887.
- [Othmane et al.(2021)] Othmane, A., Rudolph, J., Mounier, H., 2021. Systematic comparison of numerical differentiators and an application to model-free control. *European Journal of Control* 62, 113–119.
- [Pan et al.(2013)] Pan, R., Natarajan, P., Piglione, C., Prabhu, M.S., Subramanian, V., Baker, F., VerSteeg, B., 2013. PIE: A lightweight control scheme to address the bufferbloat problem, in: *IEEE 14th International Conference on High Performance Switching and Routing (HPSR)*, pp. 148–155.
- [Rabiner and Gold(1975)] Rabiner, L.R., Gold, B., 1975. *Theory and Application of Digital Signal Processing*. Prentice-Hall, Englewood Cliffs.
- [Riley and Henderson(2010)] Riley, G.F., Henderson, T.R., 2010. The *NS-3* network simulator. In: Wehrle, K., Güneş, M., Gross, J. (Eds): *Modeling and Tools for Network Simulation*, pp. 15–34, Springer, Berlin.
- [Robinson(1974)] Robinson, A., 1974. *Non-standard Analysis* (2nd ed.). North-Holland, Amsterdam.
- [Ryu et al.(2005)] Ryu S., Ryu B., Jeong, Park S., 2005. PI-PD controller for adaptive and robust active queue management for Internet congestion control. *Simulation* 81, 437–459.
- [Smith(1957)] Smith, O.J.M., 1957. Closer control of loops with dead time. *Chemistry Engineering Progress* 53, 217–219.
- [Sun et al.(2003)] Sun J., Chen G., Ko K.-T., Chan S., Zukerman M., 2003. PD-controller: a new active queue management scheme, *GLOBECOM'03*, San Francisco, 3103–3107.

- [Varma(2015)] Varma, S., 2015. Internet Congestion Control. Morgan Kaufmann, Waltham.
- [Villagra and Herrero-Perez(2012)] Villagra, J., Herrero-Perez, D., 2012. A comparison of control techniques for robust docking maneuvers of an AGV. IEEE Trans. Control Systems Technology 20, 1116–1123.
- [Xu et al.(2015)] Xu Q., Li F., Sun J., Zukerman M., 2015. A new TCP/AQM system analysis. J. Network Computer Applications 57, 43–60.
- [Yosida(1984)] Yosida, K., 1984. Operational Calculus (translated from the Japanese). Springer, New York.